

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

Final Examination (2 hours 30 minutes)

Mathematics 142

Fall 2019

Instructions: Answer all the questions. Show all work.

1. Let $f(x) = x^5 + x^3 + 7x$.
 - a) Show that $f(x)$ is a one-to-one function and therefore has an inverse function.
 - b) Find $(f^{-1})'(-9)$.

2. Find $\frac{dy}{dx}$ for each of the following. Algebraic simplification is not necessary.
 - a) $y = \tan^{-1}(e^{3x}) + e^{-3x}$
 - b) $y = \frac{\ln(x^2 + 1)}{\sec(e^x)}$
 - c) $y = \int_{\sin^{-1} x}^0 \sqrt{e^t + 1} dt$
 - d) $y = \frac{(x - 9)^5 e^{17x}}{2x^2 + 4}$ (hint: use logarithmic differentiation)

3. Use the definition of the integral as the limit of a Riemann sum to evaluate $\int_0^3 (x^2 - 1) dx$.
(**Note:** $\sum_{i=1}^n i = \frac{n(n+1)}{2}$; $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$)

4. Find the following integrals:
 - a) $\int \frac{\sin x}{1 + \cos^2 x} dx$
 - b) $\int \frac{2 \ln x}{x[1 + (\ln x)^2]} dx$
 - c) $\int \frac{x^3 - 2x + 7\sqrt{x}}{x^{3/2}} dx$
 - d) $\int_0^{\pi/4} \frac{\sec^2 x}{e^{\tan x}} dx$

5. Let R be the region in the plane bounded by the curves $y = 4x - x^2$ and $y = 2x$.
 - a) Find the area of R .
 - b) Find the volume of the solid generated by rotating R about
 - i) the x -axis
 - ii) the line $x = 2$.

6. Find the length of the curve $y = \frac{1}{6}(x^2 + 4)^{3/2}$ where $0 \leq x \leq 3$.

7. Solve the differential equation $\frac{2y}{\sqrt{x^3 + 1}} \cdot \frac{dy}{dx} = 3x^2$ if $y = 4$ when $x = 2$.

8. The half-life of Thorium-230 is 8000 years. Suppose we have an initial sample of 90 grams.
 - a) Find a formula that can be used to compute the mass remaining after t years.
 - b) How much of this sample will remain in 10,000 years? (Round your answer to the nearest hundredth.)
 - c) In how many years will the sample decay to 50 grams? (Round your answer to the nearest hundredth.)