Final Examination
2.5 hours

Mathematics 142   Spring 2016
Instructions: Answer all questions.   Show all work.

1.   a) Evaluate the Riemann sum for $f(x) = 3x - x^2$ on $[0,6]$ using 3 equal subintervals and midpoints.
      b) How much different is the approximation in part (a) from the actual area beneath the curve $y = f(x)$ from $x = 0$ to $x = 6$?

2.   Evaluate each of the following definite and indefinite integrals. Write each answer to a definite integral as an exact value, not as a decimal approximation.

   a) $\displaystyle\int 18x^2 \cos(2x^3 + 4)\, dx$

   b) $\displaystyle\int \frac{(8x + 12)}{(2x^2 + 6x)^{\frac{1}{2}}}\, dx$

   c) $\displaystyle\int_0^{\frac{\pi}{6}} (2e^{3x} + 3\sec^2 x)\, dx$

   d) $\displaystyle\int \sec^3 x \tan x\, dx$

3.   Find the derivative of each of the following functions: (Algebraic simplification unnecessary.)

   a) $y = 3x^x \tan(e^{2x} - 4x^6)$

   b) $y = \tan^{-1}(4x^7)$

   c) $y = \ln\left(\dfrac{3x + 5}{x^3 + 4}\right)^{\frac{1}{3}}$

   d) $y = \displaystyle\int_{x^3}^4 e^{(2t+5)^{\frac{1}{2}}}(4t^5)\, dt$

4.   Find f_{ave}, the average value of the function $f(x) = \dfrac{1}{1 + x^2}$ on the interval $[\frac{\sqrt{3}}{3}, \sqrt{3}]$.

5.   a) Use the method of disks to find the volume generated by rotating the region bounded by the graphs of $y = x + 7$, $y = 0$, $x = 3$ about $y = -1$.

      b) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the graphs of $y = -3x^2 + 12x + 36$ and $y = 0$ about the line $x = -2$.

6.   Solve differential equation $\dfrac{dy}{dx} = \dfrac{1 + x}{xy}$ for y, given that $y = 2$ when $x = 1$.

(continued on the back)

7. Mike placed a bottle of soda on his desk and left it there until its temperature was the same as room temperature which was 72 degrees Fahrenheit. He took the bottle of soda with him outside where the temperature was 37 degrees Fahrenheit. After 45 minutes, the temperature of the soda bottle was 48 degrees Fahrenheit.
    a) What was the temperature of the soda bottle after 15 minutes?
    b) How long would it take for the soda bottle to cool to 40 degrees Fahrenheit?

8. Determine the length of the arc of the curve given by $x = \left(\frac{2}{15}\right)(5y - 1)^{\frac{3}{2}}$, where $1 \le y \le 16$.

9. The acceleration of a particle is given by the function $a(t) = 30t^4 + 28t^6 - 6t + 5$, where the position function $s(t)$ has values $s(0) = 4$ and $s(1) = 10$. Find, $v(t)$ and $s(t)$, the velocity and position functions of the particle.

10. Let $f(x) = 3x^3$.
    a) Show that f is one to one.
    b) Find f^{-1}.
    c) Calculate $(f^{-1})'(x)$ and state the domain and range of $(f^{-1})'$.
    d) Calculate $(f^{-1})'(24)$.