

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2.5 Hours

Math 142

Spring 2017

Please answer all nine questions in the blue book provided, and show all your work.

1. Evaluate each of the following expressions. (For some, the answer is a single number; for others, it is a function. When it is a number, please give an exact answer, not just an estimate to a few decimal places.)

(a) $\int_{-1}^2 (3x^3 - 5x + 10) dx.$

(b) $\int \frac{t dt}{t^2+5}.$

(c) $\frac{d}{dx} \int_x^4 e^{\sqrt{3t^3}} dt.$

(d) $\int \frac{4}{\sqrt{1-x^2}} dx.$

(e) $\int_1^3 x^3 \sqrt{e^{2x^4+6}} dx.$

- (f) (Suggestion: use logarithmic differentiation.)

$$\frac{d}{dx} \left(\frac{x^x \cdot \sqrt{x^2 + x + 5}}{(x^2 + 3)} \right).$$

2. (a) Write out a Riemann sum for the function $f(x) = 6 \cos x$ on the interval $[0, \pi]$, using five equal subintervals. You may pick any sample points you like from those subintervals, but be sure to say what they are. (Don't just call them x_1^* , x_2^* , etc.)
- (b) Evaluate the following limit. You may use any method you like, but you must say why it works: show your algebraic work, and name any theorems you use.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{1}{2} \left(\frac{3i}{n} + 1 \right) \cdot \frac{3}{n} \right).$$

3. Show that the function $h(x) = 2x - \cos(x) + 6$ has an inverse function, and give the domain, the range, and the derivative of the inverse function. (You may express your answer in terms of x or $h^{-1}(x)$.)

Exam continues on back.

4. Set up a definite integral that is equal to the length of the curve $(y - x)^2 = x^3 + 9$ between the points $(0, -3)$ and $(3, -3)$. You do not need to find the exact length as a number; just giving the integral will suffice. But be careful about solving the equation for y .
5. Find all possible solutions to the differential equation

$$\frac{dy}{dx} + x^2 \cdot \frac{dy}{dx} = y \cdot \ln(y).$$

(Finding one solution will get you most of the credit for this problem, but to be perfect, you must name all possible solutions.)

6. An architect designs a glass dome to be constructed at the top of a building. To form the dome, she revolves the curve $y = 16 - x^2$ (from $y = 0$ to $y = 16$) around the y -axis to be the outer edge of the dome. Then she revolves the line $y = 16 - 4x$ (from $y = 0$ to $y = 16$) around the y -axis to form the inner edge of the dome. The dome itself is formed by revolving the region between these two curves around the y -axis. Find the total volume of glass that will be required to construct this dome. (All measurements here are in meters.)
7. A sample of 100 mg of the radioactive isotope tritium-3 decayed to 94.5 mg after a year.
- What is the half-life of tritium-3?
 - How long would it take for the sample to decay to 20% of its original amount?
 - At what rate is the mass of the sample decreasing 2 years after the experiment began? (Include units in your answer.)
8. A particle moves along the x -axis in such a way that its velocity at time t is given by $v(t) = 4 - 3t^2$.
- Find the total displacement of the particle from time $t = 1$ to $t = 3$.
 - Find the total distance traveled by the particle from time $t = 1$ to $t = 3$.
 - Find an equation for the position of the particle at time t , if its position at time 1 is the x -coordinate 14.
9. Find y' if

$$\tan^{-1}(3x) = \sin^{-1}(2y).$$

The expressions \tan^{-1} and \sin^{-1} here denote inverse trigonometric functions, also known as arctan and arcsin.

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