QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 142 — Spring 2018

**Instructions**. Answer each question in the blue book. Show your work and justify your answers.

1. Find the derivative $\frac{dy}{dx}$. Algebraic simplification not required.

   a. $y = \cos^{-1}(1/x)$    b. $y = e^{\sqrt{x}} \sin^{-1}(e^{2x})$    c. $y = 5^{(x^2)} + ln(ln(x^3))$

   d. $y = ln(1 + x^3)\ arctan(x^2)$    e. $y = (x^2 + 6)^x$    f. $y = \int_0^{x^3} \cos(t^2)dt$

2. Ten kilograms of Carbon-14 is placed in a box. If the half-life of Carbon-14 is 5700 years, how many kilograms will remain after 100 years? Round your answer to three decimal places.

3. Find each of the following integrals:

   a. $\int \frac{x}{\sqrt{5 - x^2}}\, dx$

   b. $\int \frac{e^{3x}}{(9 + e^{6x})}\, dx$

   c. $\int \frac{dx}{x\ (2 + ln(x))}$

   d. $\int_0^{\pi/2} \frac{\sin(x)\ dx}{e^{\cos(x)}}$

4. Let $R$ be the region enclosed by the graphs of $y = x^3$ and $y = x$.

   a. Sketch the region $R$. Take care to find ALL intersections of the two curves.

   b. Compute the area of $R$.

   c. Compute the volume of the solid obtained by rotating $R$ about the $x$-axis.

   d. Set up but do not evaluate the integral that can be used to compute the volume of the solid generated by rotating $R$ about the line $y = -2$.

5. Assuming $t > 0$, solve the differential equation $\frac{dy}{dt} = \frac{1+t}{ty}$, where $y(1) = -4$.

6. a. Evaluate the integral $\int_1^3 (x^2 + 1)dx$

   b. Compute this same integral as a limit of Riemann sums. You can use $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ and $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, as needed.

   c. Find the average value of $f(x) = x^2 + 1$ on the interval $[1, 3]$.

7. Let $f(x) = x^3 + 3x + 2$

   a. Using a suitable computation, show that f has an inverse function.

   b. Find $f^{-1}(6)$ and the derivative $(f^{-1})'(6)$

8. If $f(x) = \frac{4\sqrt{2}}{3}x^{3/2} - 1$, compute the length of the arc of the graph of $f$ between the points corresponding to $x = 0$ and $x = 1$.