

QUEENS COLLEGE
Department of Mathematics
Final Examination
2½ Hours

Mathematics 142

Spring 2019

Instructions. Answer each question in the blue book. Show your work and justify your answers.

1. Find the derivative $\frac{dy}{dx}$ of the following functions. Simplify if possible.

a) $y = \int_{1-2x}^3 \sin t^2 dt$

b) $y = \tan^{-1}(\ln x)$

c) $y = e^{\tan x}$

d) $y = \arcsin(\cos x)$

e) $y = 5^{x^2-3}$

2. Using the inequality $\tan x \geq x$ for $0 \leq x < \frac{\pi}{2}$, show, without using your calculator, that

$$\int_{0.1}^{0.5} \tan x dx \geq 0.12.$$

Clearly state the theorem that you are using.

3. Find each of the following:

a) $\int \frac{1}{1+9x^2} dx$

b) $\int \frac{1}{(21x+13)^4} dx$

c) $\int_{\pi/6}^{\pi/3} \frac{\sec^2 x}{\sqrt[3]{\tan^5(x)}} dx$

d) $\int_0^3 \frac{x}{e^{x^2}} dx$

e) $\int \tan x dx$

4. Using the definition of the definite integral as a limit of a Riemann sum, compute

$$\int_{-3}^1 (x^2 - 2x) dx$$

$$(Hint: \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{and} \quad \sum_{i=1}^n i = \frac{n(n+1)}{2})$$

5. Solve the differential equation $y'y^2 = e^{y^3}x$, given that $y(0) = 1$.

6. Find the arc length of the curve $y = \frac{1}{3} + \frac{1}{9}\sqrt{x^3}$ for $0 \leq x \leq 36$.

7. a) Find the volume of the solid obtained by rotating the graph of $y = e^{x/3}$ for $0 \leq x \leq \ln 2$ about the line $y = -1$.

b) Find the volume of the solid obtained by rotating the graph of $y = e^{x^2/3}$ for $0 \leq x \leq 2$ about the line $x = -1$.

8. An amount of \$10,000 is borrowed at an interest rate of 4.25%. How much is due at the end of 5 years if the interest is compounded

a) quarterly?

b) continuously?