

QUEENS COLLEGE  
DEPARTMENT OF MATHEMATICS

Final Examination

$2\frac{1}{2}$  Hours

Mathematics 142

SPRING 2022

**Instructions:** Answer all questions. Show all work.

1. (a) (i) Show that the function  $f(x) = \sqrt{9 - x^2}$  is one-to-one on the interval  $[0,3]$ .  
(ii) Find the rule that defines  $f^{-1}$ .  
(b) If  $g(x) = 4x^3 + 2 \sin x + 2 \cos x$ , find  $(g^{-1})'(2)$ .
  
2. Let R be the region in the plane enclosed by the graphs of  $y = \frac{1}{4}x^2$  and  $y = 2\sqrt{x}$ .  
(a) Compute A, the area of R.  
(b) Compute V, the volume of the solid obtained when R is rotated about the y-axis.
  
3. The area of the region in the plane bounded above by the graph of  $y = xe^{-x^2}$ , below by the x-axis, for  $0 \leq x \leq 2$  is given by  $\int_0^2 xe^{-x^2} dx$ .  
(a) Set up a Riemann sum with four subintervals of equal length, choosing the midpoints of the subintervals, to estimate the area in question.  
(b) Compute the actual area of the region.
  
4. Find the length of the arc of the graph of  $y = \ln(\sec x)$ , where  $0 \leq x \leq \frac{\pi}{3}$ .
  
5. A stone is dropped from the top of a tower that is 200 meters high. (Assume that the only force acting on the stone is that of gravity and the acceleration due to gravity is  $-9.8 \text{ m/sec}^2$ .)  
(a) Find the height of the stone,  $h(t)$ , at time  $t$ .  
(b) How long does it take for the stone to reach the ground? (Round your answer to two decimal places.)  
(c) With what velocity does the stone strike the ground? (Round your answer to two decimal places.)  
(d) If the stone were thrown downward with a speed of  $7 \text{ m/sec}$ , how long would it take to reach the ground? (Round your answer to two decimal places.)
  
6. In each of the following, find  $\frac{dy}{dx}$ . (Algebraic simplification is unnecessary.)  
(a)  $y = \int_1^{\cos x} (2 + v^2)^9 dv$   
(b)  $y = e^{\tan \sqrt{3x}}$   
(c)  $y = \sin^{-1}(x^3) + \cot^{-1}\left(\frac{1}{x}\right)$   
(d)  $y = (\ln x)^{\cos 4x}$

(continued on the back)

7. WITHOUT USING YOUR CALCULATOR, evaluate each of the following definite integrals:

(a) 
$$y = \int_0^{\pi/4} \frac{3 + 7 \cos^2 \theta}{\cos^2 \theta} d\theta$$

(b) 
$$y = \int_1^{64} \frac{2 + \sqrt[3]{x}}{\sqrt{x}} dx$$

(c) 
$$y = \int_0^{3\pi/2} |\sin x| dx$$

8. Find each of the following indefinite integrals:

(a) 
$$\int x^9 \sin(x^{10}) dx$$

(b) 
$$\int \frac{1}{t^4 e^{t^{-3}}} dt$$

(c) 
$$\int \frac{z^4}{\sqrt[3]{2 + z^5}} dz$$

9. (a) Solve the differential equation  $y' = \frac{3 \sin x}{\cos y}$ , given that  $y = \frac{\pi}{2}$  when  $x = 0$ .

(b) The graph of the function  $y = f(x)$  passes through the point  $(1, e)$  and has the property that the slope of the tangent line to the graph at any point P is equal to twice the y-coordinate of P. Find  $f(x)$ .