Queens College
Department of Mathematics

Final Examination
$2\frac{1}{2}$ hours

Mathematics 143 Fall 2015

Show your work. You may not use any TI calculator above the number 86, the TI inspire, nor any calculator that does indefinite integrals on this exam.

1. Integrate each of the following:

(a) $\displaystyle\int \frac{\sin^2(\ln x)}{x}dx$

(b) $\displaystyle\int x\sin 3x\,dx$

(c) $\displaystyle\int \frac{5x^2+17x+25}{x(x^2+4x+5)}dx$

(d) $\displaystyle\int x^3\sqrt{9+x^2}dx$

2. Determine the convergence or divergence of

(a) $\displaystyle\int_0^1 \frac{2x}{x^2-1}dx$

(b) $\displaystyle\int_1^\infty \frac{e^{-1/x}}{x^2}dx$

3. Find the limits of the following sequences. If a sequence diverges, say so. Justify your answer without reference to the calculator.

(a) $\displaystyle\left\{\frac{(2n)!\,n^2}{(2n+2)!}\right\}$

(b) $\displaystyle\left\{\left(1+\frac{3}{n}\right)^n\right\}$

(c) $\displaystyle\left\{\frac{(-1)^n n}{n^4+3}\right\}$

(d) $\displaystyle\left\{\frac{\sin 3n}{n^3}\right\}$

4. Find the interval of convergence of

$$\sum_{n=1}^{\infty}\frac{(x+2)^n}{\sqrt{n}}\ .$$

5. Does the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n^3+4}$ converge conditionally, absolutely, or not at all?

Use an appropriate test to justify your answer.

(continued on the back)

6. Determine the convergence or divergence of each of the following series. Justify your answer in each case.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{1/n}}{n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n^4 - 1}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n \sin n}{n!}$

7. (a) Write a power series in x for $f(x) = \frac{4}{1+x^2}$.

(b) Use the power series you obtained in part (a) to find a power series in x for $g(x) = x \tan^{-1} x$.

8. Find the exact sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n e^{2n}}{(2n)!}$.

9. Using R_n, estimate the maximum error made if one uses the second Taylor polynomial of $f(x) = \sqrt{x}$ near 9 to estimate $f(x)$ on $[8.7, 9.1]$.

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