

QUEENS COLLEGE
MATHEMATICS DEPARTMENT

FINAL EXAMINATION
2 1/2 Hours

Mathematics 143

Fall 2016

Instructions:

Answer all questions.

Show all work.

1. Find the following integrals:

(a) $\int_0^{\pi/3} t \cdot \cos(2t) dt$

(b) $\int \cos x \cdot \cot^2 x dx$

(c) $\int \frac{dx}{x\sqrt{1+x^2}}$

2. Write the form of the partial decomposition of the given function. Do not determine the numerical values of the coefficients.

$$\frac{1}{(x^2 - 1)(x - 1)(x^2 + 4x + 4)(x^2 + 2x + 4)^2}.$$

3. Determine whether each integral is convergent or divergent.

(a) $\int_{-1}^2 \frac{1}{\sqrt[5]{1-x}} dx$

(b) $\int_{-\infty}^{\infty} \frac{1}{x^2+4x+6} dx$

4. Determine whether each sequence $\{a_n\}$ is convergent or divergent. If it converges, find its limit.

(a) $a_n = \frac{\tan^{-1}(n^2+1)}{n^{1/3}}$

(b) $a_n = (-1)^n \cdot \frac{n^4}{n^3-17}$

(continued on the back)

5. Determine whether each series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty} \frac{n^3 - n + 12}{n^4 + n^3 + 1}$

(b) $\sum_{n=1}^{\infty} \frac{5^n - n}{n!}$

(c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(d) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{10^{1/n}}$

6. For what values of x does the series

$$\sum_{n=1}^{\infty} (\log_2 x)^n$$

converge?

7. Let $f(x) = x^3 e^{x^2}$.

(a) Find the Maclaurin series for $f(x)$ and its radius of convergence.

(b) Find $f^{(7)}(0)$.

8. Use l'Hospital's Rule to find the limits.

(a) $\lim_{x \rightarrow 0} \frac{x^2}{e^{3x} - e^{2x}}$

(b) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{e^x}\right)^x$

9. Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$$

by expressing the n -th partial sum s_n as a telescoping sum.

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