Mathematics 143   **Fall 2016**

<u>**Instructions:**</u>   **Answer all questions.**   <u>**Show all work.**</u>

1. Find the following integrals:

(a) $\int_0^{\pi/3} t \cdot \cos(2t)dt$

(b) $\int \cos x \cdot \cot^2 x \, dx$

(c) $\int \frac{dx}{x\sqrt{1+x^2}}$

2. Write the form of the partial decomposition of the given function. Do not determine the numerical values of the coefficients.

$$\frac{1}{(x^2-1)(x-1)(x^2+4x+4)(x^2+2x+4)^2}.$$

3. Determine whether each integral is convergent or divergent.

(a) $\int_{-1}^{2} \frac{1}{\sqrt[5]{1-x}} dx$

(b) $\int_{-\infty}^{\infty} \frac{1}{x^2+4x+6} dx$

4. Determine whether each sequence $\{a_n\}$ is convergent or divergent. If it converges, find its limit.

(a) $a_n = \frac{\tan^{-1}(n^2+1)}{n^{1/3}}$

(b) $a_n = (-1)^n \cdot \frac{n^4}{n^3-17}$

5. Determine whether each series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty} \frac{n^3 - n + 12}{n^4 + n^3 + 1}$

(b) $\sum_{n=1}^{\infty} \frac{5^n - n}{n!}$

(c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(d) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{10^{1/n}}$

6. For what values of x does the series
$$\sum_{n=1}^{\infty} (\log_2 x)^n$$
converge?

7. Let $f(x) = x^3 e^{x^2}$.

(a) Find the Maclaurin series for $f(x)$ and its radius of convergence.

(b) Find $f^{(7)}(0)$.

8. Use l'Hospital's Rule to find the limits.

(a) $\lim_{x \to 0} \frac{x^2}{e^{3x} - e^{2x}}$

(b) $\lim_{x \to \infty} \left(1 + \frac{1}{e^x}\right)^x$

9. Find the sum of the series
$$\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$$
by expressing the n-th partial sum s_n as a telescoping sum.