QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 143 **Fall 2018**

Instructions: Answer all the questions. Show all work.

1. Evaluate the following limits:

   a) $\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$

   b) $\lim_{x\to\infty}(e^x+x)^{1/x}$

2. a) Evaluate the following integrals:

   i) $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

   ii) $\int_0^{\pi/2} \sin^3 x\cos^2 x\,dx$

   iii) $\int \frac{x^2-x+6}{x^3+3x}\,dx$

   b) Determine whether the integral is convergent or divergent. If it is convergent, find its value.

   i) $\int_2^{\infty} \frac{dx}{x\ln x}$

   ii) $\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$

3. Find the sum of the series:

   a) $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{5^n}$

   b) $\sum_{n=1}^{\infty}[\tan^{-1}(n+1)-\tan^{-1}(n)]$

   c) $1-\ln 2+\frac{(\ln 2)^2}{2!}-\frac{(\ln 2)^3}{3!}+\cdots$

4. a) Find a power series representation for $f(x)=\ln(1+x)$ and find its radius of convergence.

   b) Show that $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)\cdot 3^n}=\frac{\pi}{2\sqrt{3}}$.

   c) Express the indefinite integral $\int x^2\tan^{-1}x\,dx$ as a power series. What is its radius of convergence?

   Hint: For 4b) and 4c) use the following power series representation:

   $$\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots \quad \text{for } |x|<1$$

(continued on the back)

5. Find the radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{2n+1}$.

6. Determine whether each of the following series is convergent or divergent. Show all details and specify which test you are using.

a) $\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}{n!}$

b) $\sum_{n=1}^{\infty} \frac{n^{2n}}{(1+2n^2)^n}$

7. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent:

a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+4}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n \tan^{-1} n}{n^2}$

8. a) Find $T_3(x)$, the third Taylor polynomial of $f(x) = \ln(1+2x)$ at $a = 1$.
   b) If $T_3(x)$ is used to approximate $f(x)$ when $0.5 \leq x \leq 1.5$, how accurate is this approximation?