Queens College
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 143 Fall 2019

Instructions:

- ***Read each problem carefully.*** Make sure you understand what the problem is asking.
- **You must show all your work.** No credit will be given to a problem without work. For incorrect solutions, partial credit will be given where appropriate.
- All your work and solutions must be recorded in the provided Blue Book. Additional Blue Books are available if necessary.
- You are allowed to use a calculator that is in accordance with the calculator policy set by the mathematics department. You may not share calculators during the exam.
- No notes or devices other than a writing utensil and calculator may be used during the exam.
- Unless otherwise noted, answers must be precise, not approximate (for example, $\frac{1}{3} \neq 0.33$).

1. For each of the following statements, decide whether it is TRUE or FALSE. (No work is required.)
   (a) L'Hôspital's rule *cannot* be used to compute $\lim_{x\to 1} \frac{x^2+1}{x+2}$.
   (b) If f and g are continuous functions with $f(x) > g(x) \geq 0$ for every $x \geq 1$ and $\int_1^\infty f(x)\,dx$ diverges, then the Comparison Theorem implies $\int_1^\infty g(x)\,dx$ diverges.
   (c) Every absolutely convergent series is convergent.
   (d) If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, then the Ratio Test guarantees that $\sum_{n=1}^{\infty} a_n$ converges.
   (e) The Alternating Series Test guarantees that the alternating series $\sum_{n=1}^{\infty} (-1)^n n$ converges.

2. Find each of the following integrals:
   (a) $\int \frac{2x+5}{x^3-2x^2+x}\,dx$
   (b) $\int_1^\infty xe^{-3x}\,dx$
   (c) $\int \frac{dx}{x^2\sqrt{x^2-4}}$
   (d) $\int \ln(x^2+1)\,dx$

3. Compute the limits of the following sequences.
   (a) $\left\{\frac{(\ln n)^2}{n}\right\}_{n=1}^{\infty}$
   (b) $\{(1+\frac{7}{n})^n\}_{n=1}^{\infty}$.

(continued on the back)

4. Compute the sum $\sum_{n=0}^{\infty} \frac{3^{2n-1}}{10^{n+1}}$

5. Determine if each of the following series is absolutely convergent, conditionally convergent, or divergent. Clearly indicate where and how any convergence/divergence tests are used in the justification of your conclusion.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{7^n}$

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{3n^2 - 2}$

   (c) $\sum_{n=1}^{\infty} \frac{2n^2 - 5}{n^2 + n + 1}$

6. Find the radius and interval of convergence of each of the following power series.

   (a) $\sum_{n=1}^{\infty} \frac{3^n x^n}{4n - 1}$

   (b) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2}$

7. In this problem you are asked to approximate the function $f(x) = \ln x$. In parts (b) and (c), your answers may be in decimal form.

   (a) Approximate the function $f(x) = \ln x$ by its second Taylor polynomial centered at $a = 1$ (do not try to find the entire Taylor series).

   (b) How accurate is this approximation when $0.8 \le x \le 1.2$? (This is asking you to find a numerical upper bound for the remainder function $|R_2(x)|$ when $0.8 \le x \le 1.2$.)

   (c) Using the Taylor polynomial found in part (a), approximate $\ln(1.1)$.