QUEENS COLLEGE
MATHEMATICS DEPARTMENT

FINAL EXAMINATION
2 1/2 Hours

Mathematics 143 **Spring 2015**

Instructions: **Answer all questions.** **Show all work.**

1. Find the following integrals:

(a) $\int \frac{t^5}{\sqrt{1-t^2}} dt$

(b) $\int_0^{\pi/3} \tan^3 x \cdot \sec^2 x dx$

(c) $\int x^2 \ln x dx$

(d) $\int \frac{5dx}{(x-1)(x^2+2x+2)}$

2. Determine whether each integral is convergent or divergent.

(a) $\int_{-1}^{0} \frac{x}{\sqrt[6]{1+x}} dx$

(b) $\int_1^{\infty} \frac{\arctan x}{(x+5)^{3/2}} dx$

3. Determine whether each sequence $\{a_n\}$ is convergent or divergent. If it converges, find its limit.

(a) $a_n = \frac{\cos n}{1+\sqrt{n}}$

(b) $a_n = (-1)^n \cdot \frac{n^2}{n^2+3}$

4. Determine whether each series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty} \frac{n+n^2}{\cos^2 n}$

(b) $\sum_{n=1}^{\infty} \frac{2}{4^n-n}$

(c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(d) $\sum_{n=1}^{\infty} \frac{9^n}{2^{n+1} \cdot 5^n \cdot n!}$

(continued on the back)

5. Find the radius of convergence and interval of convergence of the series

$$\sum_{n=2}^{\infty} \frac{(x-1)^n}{n^{3/2} \cdot \ln n}.$$

6. Use l'Hospital's Rule to find the limits.

(a) $\lim_{x\to\infty} \frac{x^2}{e^{3x}+x^3+1}$

(b) $\lim_{x\to 0^+} (\cot x)^{\sin x}$

7. Find the Maclaurin series for $f(x) = \frac{x}{x^2+1}$ and its radius of convergence.

8. Determine the least number of terms of the Maclaurin series for $\sin x$, so that the error in calculating $\sin 2$ does not exceed 0.001.