Math 143   Spring 2016

Instructions: *Answer all questions. Show all work.*

1. Evaluate each of the following integrals:

   (a) $\displaystyle\int \frac{\ln(x)}{\sqrt{x}}\,dx$

   (b) $\displaystyle\int x^3\sqrt{x^2+4}\,dx$

   (c) $\displaystyle\int \frac{dx}{(x-1)^2(x^2+4)}$

2. Determine whether $\displaystyle\int_{-\infty}^{0} xe^{2x}\,dx$ converges or diverges. Evaluate it if it is convergent.

3. Find the limit, if it exists.

   (a) $\displaystyle\lim_{x\to 0}\frac{\sin(5x)}{\tan(4x)}$

   (b) $\displaystyle\lim_{x\to\infty}(2x)^{1/3x}$

4. Determine if each series is absolutely convergent, conditionally convergent, or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{3n-10}{2n+3}$

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{n^4(-3)^n}{(n+2)!}$

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{3n}}{n^3+5}$

   (d) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt[3]{n}}$

5. Find the radius and interval of convergence of the following power series:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-4)^n}{n+1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n! x^n}{3^n}$

6. Find a power series in x for $F(x) = \displaystyle\int x^4 e^{x^2} \, dx$ and state its radius of convergence.

7. Find the first three terms of the Maclaurin series for

(a) $\dfrac{1}{1-x^3}$

(b) $e^{-x}\sin(x)$

8. Let $f(x) = \frac{3}{2+x}$.

(a) Find the second Taylor polynomial of f about $a = 1$.

(b) Approximate $f(1.3)$ using the Taylor polynomial in part (a).

(c) Suppose that x is in the interval $(0, 2)$. First obtain a general expression for the error term $R_2(x)$, with $a = 1$. Then find a bound for this error, where x is in the interval $(0, 2)$.