Queens College Mathematics Department Final Examination $2\frac{1}{2}$ hours

Math 143

Spring 2016

Instructions: Answer all questions. Show all work.

1. Evaluate each of the following integrals:

(a)
$$\int \frac{\ln(x)}{\sqrt{x}} dx$$

(b)
$$\int x^3 \sqrt{x^2 + 4} dx$$

(c)
$$\int \frac{dx}{(x-1)^2 (x^2 + 4)}$$

- 2. Determine whether $\int_{-\infty}^{0} x e^{2x} dx$ converges or diverges. Evaluate it if it is convergent.
- 3. Find the limit, if it exists.

(a)
$$\lim_{x \to 0} \frac{\sin(5x)}{\tan(4x)}$$

(b)
$$\lim_{x \to \infty} (2x)^{1/3x}$$

4. Determine if each series is absolutely convergent, conditionally convergent, or divergent.

(a)
$$\sum_{n=1}^{\infty} \frac{3n-10}{2n+3}$$

(b) $\sum_{n=1}^{\infty} \frac{n^4(-3)^n}{(n+2)!}$
(c) $\sum_{n=1}^{\infty} \frac{\sqrt{3n}}{n^3+5}$
(d) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt[3]{n}}$

(continued on the other side)

5. Find the radius and interval of convergence of the following power series:

(a)
$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-4)^n}{n+1}$$
 (b) $\sum_{n=0}^{\infty} \frac{n! x^n}{3^n}$

6. Find a power series in x for $F(x) = \int x^4 e^{x^2} dx$ and state its radius of convergence.

7. Find the first three terms of the Maclaurin series for

(a)
$$\frac{1}{1-x^3}$$
 (b) $e^{-x}\sin(x)$

8. Let $f(x) = \frac{3}{2+x}$.

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- (a) Find the second Taylor polynomial of f about a = 1.
- (b) Approximate f(1.3) using the Taylor polynomial in part (a).
- (c) Suppose that x is in the interval (0, 2). First obtain a general expression for the error term $R_2(x)$, with a = 1. Then find a bound for this error, where x is in the interval (0, 2).

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