

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
 $2\frac{1}{2}$ HOURS

Mathematics 143

Spring 2017

Answer all questions and show all work.

1. Evaluate each of the following limits (Answers could be $\pm\infty$ or DNE.)

a) $\lim_{x \rightarrow 0} \frac{2e^x - x^2 - 2x - 2}{\sin x - x}$

b) $\lim_{x \rightarrow 0} \left(1 + \frac{x}{4}\right)^{\frac{2}{x}}$

2. Evaluate each of the following integrals: (Calculator solutions will not be accepted.)

a) $\int e^{\sqrt{x}} dx$

b) $\int \frac{2x^3 + x^2 + 9}{x^4 + 9x^2} dx$

c) $\int \frac{1}{\sqrt{x^2 + 16}} dx$

d) $\int_0^1 \frac{1}{x^2 - 2x + 2} dx$

e) $\int_0^{\pi/2} \sin^2 x \cos^3 x dx$

3. Use an appropriate test to determine if each of the following series converges or diverges. Show all details and specify which test you are using.

a) $\sum_{n=1}^{\infty} \frac{n(n+1)}{(n+2)(n+3)}$

b) $\sum_{n=0}^{\infty} \frac{2^n}{3^n + 4^n}$

c) $\frac{3}{e} + \frac{5}{e^2} + \frac{7}{e^3} + \frac{9}{e^4} + \dots$

4. Let $s_n = \sum_{k=1}^n a_k$

a) If $a_k = \frac{6}{(k+1)(k+2)}$ find s_n and determine the value of $\sum_{k=1}^{\infty} a_k$.

b) If $s_n = \frac{n}{n+1}$ find a_k and determine the value of $\sum_{k=1}^{\infty} a_k$.

(continued on other side)

5. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2 + 1}$ converges absolutely, converges conditionally, or diverges.
6. Find the first four terms of the Maclaurin series for the function $f(x) = \sqrt{4+x}$ and use it to approximate $\sqrt{5}$.
7. Let $f(x) = \sum_{n=0}^{\infty} \frac{1}{(n+1)3^n} x^{n+1} = x + \frac{x^2}{6} + \frac{x^3}{27} + \frac{x^4}{108} + \dots$
- Determine the interval of convergence of this series.
 - Use your knowledge of series to compute $f'(2)$.
8. Use series to approximate $\int_0^1 x^3 \sin(x^2) dx$, accurate to 4 decimal places.
- (Hint: Recall that $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$.)

(Answers which are not accurate to 4 decimal places will not be given full credit.)

This material is the property of Queens College and may not be reproduced in whole or in part, for sale or free distribution, without the written consent of Queens College, Flushing, NY 11367.