QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 143 **Spring 2018**

<u>Instructions:</u> <u>Answer all the questions.</u> <u>Show all work.</u>

1. Evaluate:

   a) $\displaystyle\int \frac{x^3}{(x^2+1)^3}\,dx$ b) $\displaystyle\int \frac{2x^2-x+4}{x^3+4x}\,dx$ c) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{\sqrt{1+\sin^2 x}}\,dx$

2. Determine if each of the improper integrals is convergent or divergent. If convergent, determine its value.

   a) $\displaystyle\int_1^{\infty} \frac{dx}{(1+x)\sqrt{x}}$ b) $\displaystyle\int_0^4 \frac{\ln x}{\sqrt{x}}\,dx$

3. Evaluate the following limit: $\displaystyle\lim_{x\to 0}\,(1-2x)^{\frac{1}{x}}$

4. Prove that if $I_n = \displaystyle\int_0^{\pi/2} \sin^n x\,dx$ then $I_n = \dfrac{n-1}{n} I_{n-2}$ (where $n \geq 2$ is an integer.)

5. Find the radius of convergence and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n\sqrt{n}}$.

6. Determine whether the series is convergent or divergent: (Show ALL work)

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{5^n\, n!}$

   b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(3n)}{1+(1.2)^n}$

   c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{2n}}{(1+2n^2)^n}$

   d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{n}$

7. a) Find a power series representation for
   i) $f(x) = \tan^{-1} x$
   ii) $f(x) = \ln(1+x)$
   State the radius of convergence in each case.

   b) Express the indefinite integral $\displaystyle\int \frac{\tan^{-1} x}{x}\,dx$ as a power series in x.

8. Find the exact sum of the following series:

   a) $\displaystyle\sum_{n=1}^{\infty}\left(e^{\frac{1}{n}} - e^{\frac{1}{n+1}}\right)$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n\,5^n}$ c) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n\, n!}$

9. a) Approximate the function $f(x) = \sqrt[3]{x}$ by a Taylor polynomial of degree 2 at $a = 8$.
   b) How accurate is this approximation when $7 \leq x \leq 9$?