QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 143 Spring 2019

Directions. *Read each question on this exam before you start working so you can get the flavor of the questions. Please show all of your work. Unsupported answers will not even be graded. Do not cheat, else you pay with your academic life.*

1. Find each of the following integrals.

   (a) $\displaystyle\int x^2 \ln(x)\, dx$

   (b) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-1}}\, dx$

   (c) $\displaystyle\int \tan^2(x)\sec^4(x)dx$

   (d) $\displaystyle\int \frac{dx}{(x-3)(x^2+1)}$

2. Determine whether each of the following integrals converges or diverges. If an integral converges, find its value.

   (a) $\displaystyle\int_0^\infty \frac{e^x}{e^{2x}+3}dx$

   (b) $\displaystyle\int_2^3 \frac{1}{\sqrt{3-x}}dx$

3. Compute each of the following limits.

   (a) $\displaystyle\lim_{x\to\infty} \frac{e^{x/10}}{x^3}$

   (b) $\displaystyle\lim_{x\to\infty} (2x)^{\frac{1}{3x}}$

4. Classify each of the following series as absolutely convergent, conditionally convergent, or divergent. Indicate the test you use to justify your conclusions.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{7n-1}{6n+4}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3(-1)^n}{5^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{5n}}{n^2+1}$

   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{\sqrt{n}}$

(continued on the back)

5. Find the radius and interval of convergence of each of the following power series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2 5^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n (x-2)^n}{(n+2)!}$

6. Find a power series for $F(x) = \displaystyle\int x^2 e^{x^3} dx$ and state its radius of convergence.

7. Find the first three terms of the Maclaurin series for

   (a) $\frac{1}{1-(2x)^4}$

   (b) $\sqrt{1+(3x)^2}$

8. If $f(x) = \frac{5}{3+x}$,

   (a) find the second Taylor polynomial of f centered about $a = 1$.

   (b) approximate $f(1.3)$ using the Taylor polynomial found in part (a).

   (c) obtain an expression for the error term, $R_2(x)$, and find a bound for this error.