

**QUEENS COLLEGE
MATHEMATICS DEPARTMENT**

Final Examination

$2\frac{1}{2}$ Hours

Math 151 **Show all work in blue books for all questions**

Fall 2015

1. Find each limit. (Answers may be $\pm\infty$, or undefined.)
 - a) $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3}$
 - b) $\lim_{x \rightarrow 3^-} \frac{x^2 + 9}{x - 3}$
 - c) $\lim_{x \rightarrow 3} \frac{\sin(x - 3)}{x^2 - 9}$
 - d) $\lim_{x \rightarrow 3} \frac{\frac{1}{3} - \frac{1}{x}}{3 - x}$
 - e) $\lim_{x \rightarrow +\infty} \frac{2x + \sin(2x)}{3x + 1}$
2. In each of the following, find $\frac{dy}{dx}$:
 - a) $y = 5x^3 - 2x^2 + \sqrt{x} - x^{-2}$
 - b) $y = (x^2 - 5x)^3(2x + x^3)^4$
 - c) $y = \frac{\sec^2(x)}{x^2 + 1}$
 - d) $\sin(x + y) = \tan(xy^2)$
 - e) $y = \int_1^{x^2} \frac{\cos(t)}{1 + t^3} dt$
3. Let $f(x) = \frac{1}{2x + 1}$. Use the definition of the derivative as a limit to find $f'(x)$.
4. Let $f(x) = \frac{10 - x}{x^2}$
 - a) Find $f'(x)$ and $f''(x)$ and simplify both.
 - b) Find all critical numbers of f .
 - c) Find the intervals on which f is increasing and those on which it is decreasing.
 - d) Find all local extrema of f .
 - e) Find the intervals where f is concave upward, where it is concave downward, and any inflection points.
 - f) Find any horizontal or vertical asymptotes.
 - g) Sketch the graph using this information. (The sketch should agree with your answers in parts a) to f).)
5.
 - a) Show, using theorems from this course, that the equation $x^7 + 3x - 5 = 0$ has exactly one real solution.
 - b) Use your calculator to find this solution, accurate to 5 decimal places.
6. A wire 2 feet long will be cut into two pieces. Each of these pieces will then be formed into a square. Find the lengths of the two pieces which minimize the total area of the two squares.
7. A 10-foot ladder leans against a vertical wall. The top of the ladder begins to slide down the wall at a rate of 2 feet/sec. How fast is the bottom of the ladder sliding away from the wall when the top of the ladder is 8 feet above the ground?
8.
 - a) Find an equation of the tangent line to the curve $y = \sqrt{1 + x}$ at the point where $x = 3$.
 - b) Use a linear approximation (differentials) to estimate $\sqrt{4.1}$.
9. Find the integrals:
 - a) $\int (x^2 + 3x)^2 dx$
 - b) $\int (1 + \sin(2x))^3 \cos(2x) dx$
 - c) $\int x\sqrt{2x + 1} dx$
10. Express $\int_1^2 (4 - x) dx$ as a limit of a Riemann sum. (Hint: $\sum_{i=1}^n i = \frac{n(n + 1)}{2}$)