QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
2.5 HOURS

Mathematics 151 **Fall 2016**
Answer all questions **Show all work**

1. Use the limit laws to compute the limits below. If the limit is $\pm\infty$ or does not exist, justify your answer.

   a) $\lim_{x\to 5} \dfrac{5-x}{x^2-25}$

   b) $\lim_{x\to \infty} \dfrac{12+x^3-7x^5}{2x+x^5-12}$

   c) $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$

   d) $\lim_{x\to 0} \dfrac{\sin 2x \cdot \sin 3x}{x^2}$

   e) $\lim_{h\to 0} \dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}$

   f) $\lim_{x\to -1^+} \dfrac{x^2-4x}{x^2-3x-4}$

2. a) State the conditions that are necessary for a function $f(x)$ to be continuous at $x=c$.

   b) Use part a) to determine whether or not the function f below is continuous at $x=2$ and $x=4$.

$$f(x)=\begin{cases} x^4+4 & \text{if } 0\le x<2 \\ 4 & \text{if } 2\le x<4 \\ \sqrt{x-4}+4 & \text{if } x\ge 4 \end{cases}$$

3. Let $f(x)=\dfrac{x^2+1}{x^2-4}$.

   a) Determine any vertical or horizontal asymptotes of the graph of f.
   b) Find $f'(x)$, $f''(x)$, and all critical numbers of f.
   c) Identify the intervals where f is either increasing or decreasing, and find any local extrema.
   d) Identify the intervals where f is either concave up or concave down, and find inflection points, if any exist.
   e) Use the information found in parts a) – d) to sketch the graph of f. Your sketch should make sense based on your results.

4. A spherical-shaped balloon is being pumped with air so that its radius is increasing at a rate of 1.4 feet per second. How fast is the volume of the balloon increasing when the diameter is 12 ft ?
   Round your answer to the nearest cubic foot per second. (Hint: The volume of a sphere of radius r is given by $V=\frac{4}{3}\pi r^3$.)

(continued on the back)

5. Compute the derivative of each of the following functions. Algebraic simplification is unnecessary.

a) $f(x) = (\cot x + \cos(5x))^2 \left(3x^2 - \frac{5}{\sqrt[3]{x}}\right)$

b) $g(x) = \tan^2\left(\frac{6}{x-5}\right)$

c) $F(x) = \int_{x^2}^{5} \frac{\sin^2 t}{5-3t^2}\, dt$

d) $y = \frac{x^{-1} - 1 - \sqrt{2x}}{(2\pi)^3 - 5x^4}$

6. Determine $\frac{dy}{dx}$ if $x^2(x^2+y^2) = y^2$. Then find an equation of the tangent line to the graph of this equation at $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

7. a) State the Intermediate Value Theorem. You may state it formally, or give a **clear description** in your own words.

b) Use the Intermediate Value Theorem to show that $f(x) = 2x^3 - 7x + 8$ must have a zero in the interval $[-3, 0]$.

c) Use your graphing calculator to determine the zero in $[-3, 0]$. Round your answer to four decimal places.

8. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

9. Evaluate the following integrals.

a) $\int \sin^2 3x \cdot \cos 3x\, dx$

b) $\int \frac{\sqrt{x} - 3x^4 + \pi x^{7.5}}{2x^2}\, dx$

c) $\int x^3 \tan(x^4) \sec(x^4)\, dx$

10. a) Evaluate the definite integral $\int_0^1 (6x^2 + 1)\, dx$.

b) Compute the value of the definite integral found in part (a) by using the limit of an appropriate Riemann sum.

Useful formula: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$