QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2.5 Hours

Math 151   **Fall 2017**

Please answer all nine questions in the blue book provided, and show all your work.

1. Without using your calculator, find **FOUR** of the following limits. (*Note: If a limit is* $+\infty$, $-\infty$, *or does not exist, state this as your answer.*)

   (a)
   $$\lim_{x \to \infty} \frac{\frac{1}{2}x^{\frac{7}{2}} + \frac{3}{2}x^{\frac{5}{2}} + \frac{5}{2}x^{\frac{3}{2}} + \frac{7}{2}x^{\frac{1}{2}}}{\sqrt{x^7 + 3x^5 + 5x^3 + 7x}}$$

   (b)
   $$\lim_{x \to -1} \frac{2}{x^2 - 1} + \frac{1}{x + 1}$$

   (c)
   $$\lim_{x \to 1} \frac{x - 1}{\sqrt{x^2 + 3} - 2}$$

   (d)
   $$\lim_{x \to 0} \frac{2 \sin(4x) - 3x}{5x}$$

   (e)
   $$\lim_{x \to -3^+} \frac{x + 2}{x^3 + 3x^2}$$

2. Find $\frac{dy}{dx}$. (Algebraic simplification unnecessary.)

   (a) $y = (\cos(x))^3 + \cos(x^3)$

   (b) $y = \frac{2x^3 - 4x^2 + 7x - 9}{(4x^5 - 3)^3}$

   (c) $3xy + 2x^2 = 5y$

   (d) $y = \cot(x + \frac{\pi}{3}) - \tan(3x)\sec(4x)$

3. Let
   $$f(x) = \begin{cases} x^2 + c & \text{if } x < -2, \\ dx - 8c & \text{if } -2 \le x < 1, \\ 5dx - 2 & \text{if } x \ge 1 \end{cases}$$

   (a) If $f(x)$ is continuous everywhere, find the values of c and d.

   (b) Find an equation of the tangent line to $f(x)$ at $x = -5$.

4. ***Use the definition of the derivative*** to find $f'(x)$ if $f(x) = \frac{2}{1-x}$.

(continued on the other side)

5. (a) Show that the equation $3x+1-\sin(x) = 0$ has only one real root. (*Justify your conclusion by stating appropriate theorems.*) Then **USE YOUR CALCULATOR** to estimate this root to the nearest thousandth.

   (b) Let $f(2) = 5$, and suppose that $f'(x) \geq 6$ for all real numbers x. What is the *largest* possible value $f(-1)$ can take on?

6. Let $f(x) = \frac{x^2}{x^2-1}$.

   (a) Find intervals on which the graph of f is increasing and intervals on which it is decreasing.

   (b) Find and classify any relative extrema of f.

   (c) Find intervals on which the graph of f is concave up and those on which it is concave down.

   (d) Find any and all points of inflection of f.

   (e) Find any and all horizontal and vertical asymptotes of the graph of f.

   (f) Based upon the results found in the previous parts, sketch the graph of f. Make sure to label all significant points.

7. A strange man wants to print glamour shots of his dog Carly and hand them out to dog talent agents. He prints a rectangular picture of his dog and attaches it to the center of a rectangular piece of paper, making sure the upper and lower margins of the paper are 2 cm each, and the right and left margins of the paper are 3 cm each. If the photo itself must have an area of 96 cm^2, find the dimensions of the smallest possible piece of paper onto which he might attach the photo.

8. An inverted conical tank with radius 6 feet and a height of 8 feet is being filled with water at a rate of 4 cubic feet per minute. How fast is the water rising when the volume of the water is $\frac{81\pi}{16}$ cubic feet? (*Note: The formula for the volume of a cone is $V_C = \frac{1}{3}\pi r^2 h$.*)

9. Find **THREE** of the following indefinite integrals.

   (a)
   $$\int \left(\frac{2\sqrt{x}+1}{3x^5}\right)dx$$

   (b)
   $$\int (3x\cos^2(3x^2+1)\sin(3x^2+1))dx$$

   (c)
   $$\int (-2x+1)^3 dx$$

   (d)
   $$\int \tan(2x)\sec(2x)dx$$

10. Evaluate
    $$\int_0^3 (3x^2 - 4x)dx$$

    as the limit of a Riemann sum.

    You should use: $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$