QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

MATHEMATICS 151 **FALL 2018**
INSTRUCTIONS: SHOW ALL WORK IN YOUR BLUE BOOK FOR ALL QUESTIONS.

1. Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$ or does not exist, explain why.

   a) $\lim_{x\to 6} \dfrac{6-x}{x^2-2x-24}$

   b) $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3}$

   c) $\lim_{\theta\to 0} \dfrac{\tan(6\theta)}{4\theta+5\sin\theta}$

   d) $\lim_{x\to\infty} \dfrac{x+3}{\sqrt{16x^2+1}}$

   e) $\lim_{x\to 3^-} \dfrac{x^2-x-6}{|x-3|}$

2. Let $f(x)=\dfrac{3x}{x+5}$. Using the definition of the derivative, find $f'(x)$.

3. In each of the following find $\dfrac{dy}{dx}$: (algebraic simplification is not necessary)

   a) $y=(x^5-2x^2+6)^4(3x^3+5x^2+9x)^6$

   b) $y=\dfrac{\sec 4\theta}{1+\tan 6\theta}$

   c) $y=\cos\left(\tan\left(\sqrt{\sin 5x}\right)\right)$

   d) $\cos(x+y)=y^4\sin x$

   e) $y=\displaystyle\int_{x^2}^{3}\sqrt[3]{1+\sin t^3}\,dt$

4. Draw a graph of a single function f having all of the following properties.

   i) $f'(x)>0$ if $0<x<1$ or $1<x<2$
   $f'(x)<0$ if $x<0$ or $x>2$

   ii) $f''(x)>0$ if $x<1$ or $x>5$
   $f''(x)<0$ if $1<x<5$

   iii) $\lim_{x\to 1^-} f(x)=\infty$ $\lim_{x\to 1^+} f(x)=-\infty$

   iv) $\lim_{x\to\infty} f(x)=1$, $f(0)=0$, $f(2)=4$, $f(5)=3$

(CONTINUED ON THE BACK)

5. At a certain instant, car A is 120 miles north of car B. Car A is traveling south at a rate of 40 mi/hr while car B is traveling east at 60 mi/hr. How fast is the distance between them changing two hours later?

6. A particle moves along a line so that its velocity at time t is $v(t) = t^2 - 2t - 15$ (measured in meters per second.)
   a) Find the displacement of the particle during the time period $1 \leq t \leq 7$.
   b) Find the distance traveled during this time period.

7. Let $f(x) = \dfrac{2x}{x^2 - 4}$.
   a) For which intervals is f increasing and for which is f decreasing?
   b) Find all the local maxima and/or local minima of f.
   c) Find any and all vertical or horizontal asymptotes of f.
   d) For which intervals is f concave up and for which is f is concave down?
   e) At what x values, if any, does f have inflection points?
   f) Sketch the graph of $y = f(x)$ using the information found in parts a) – e).

8. A box with a square base and a closed top has a volume of 3,000 cubic feet. The top costs $4 per square foot, the bottom costs $ 8 per square foot and the sides cost $ 2 per square foot. Use calculus to find the dimensions of the box that will minimize its total cost of construction.

9. Find each of the following integrals:
   a) $\displaystyle\int (u + 3)(u + 2)\, du$
   b) $\displaystyle\int \sec^6 x \tan x \, dx$
   c) $\displaystyle\int x(3x + 2)^6 \, dx$

10. Evaluate $\displaystyle\int_0^3 (x^2 + 2x)\, dx$ as the limit of a Riemann sum.

    Note: $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$