QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

MATHEMATICS 151 **FALL 2019**

<u>INSTRUCTIONS</u>: SHOW ALL WORK FOR ALL QUESTIONS IN ORDER TO RECEIVE MAXIMUM CREDIT.

1) Use algebraic methods and limit laws to find each of the following limits. If a limit is $+\infty$ or $-\infty$, explain why (Note: The use of L'Hospital's Rule is not permitted).

a) $\lim\limits_{x \to +\infty} \left[\dfrac{3x^3 + 2\sqrt{x} + x}{7 - 2x^2 + 18x^3}\right]$

b) $\lim\limits_{x \to 5^-} \left[\dfrac{x^3 - 4x}{x^2 - 3x - 10}\right]$

c) $\lim\limits_{x \to \infty} \left[x - \sqrt{x^2 + 4}\right]$
(Hint: Use a conjugate expression.)

d) $\lim\limits_{x \to 0} \left[\tan(3x) + \sqrt[5]{x^4 - 32}\right]$

---

2) Let $F(x) = \frac{5}{x+1}$

a) <u>Using the definition of the derivative</u>, find $F'(x)$.

b) Write equations of the tangent lines to the graph of F that are parallel to the line given by the equation $4y + 5x = -4$.

---

3) In each of the following, find $\dfrac{dy}{dx}$ $or\ f'(x)$ (Basic arithmetic simplification is required.)

a) $y = x^3 \tan^2(\sqrt{4x}) - \pi \sin\left(\frac{1}{x}\right)$

b) $f(x) = \left(\frac{1 - \cos(kx)}{1 + \cos(kx)}\right)^n$
where n and k are positive integers

c) $x^3 + y^3 = 4xy + 2$

d) $f(x) = \displaystyle\int_{\sqrt{x}}^{-1} \frac{\sec(t^2)}{t^2 - t}\, dt$

e) $y = \dfrac{-5x^6 - \frac{2}{\sqrt{x}} + \sqrt[3]{x} + x}{2x}$

---

4) Find the second derivative for the function given by $f(x) = \sqrt{2x^2 + 8x + 4}$.

---

5) Choose EITHER question (A) or question (B), not both:

A) An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from each of the four corners and folding the up the sides. In order to obtain a box with the largest possible volume, what must be the dimensions of the squares that should be cut out from the corners of the cardboard?

B) Two cars start moving from the same point at the same time. One car travels south at 60 miles per hour, and the other travels west at 25 miles per hour. At what rate is the distance between the cars increasing two hours after they begin to move?

(CONTINUED ON THE BACK)

6) Consider the function given by $g(x) = \sqrt[3]{3x^2 - 13x - 10}$.

a) Explain why the function g must be continuous on $(-\infty, +\infty)$.

b) Compute $g'(x)$, and use the definition of continuity to either prove or disprove that g' is continuous on $(-\infty, +\infty)$.

---

7) Let $g(x) = \frac{x-1}{2x^2}$

a) Find the intervals on which g is increasing and the intervals on which g is decreasing.

b) Find the points on the graph of g (if any) where g has a local minimum and the points (if any) where g has a local maximum.

c) Find the intervals on which the graph of g is concave up and the intervals on which the graph of g is concave down.

d) Find the coordinates of any and all inflection points of the graph of g.

e) Using the information found in parts a) – d), draw a reasonable sketch the graph of g. Label all appropriate points and any asymptotes the graph of g may have.

---

8) Consider the definite integral

$$\int_0^4 (x^2 + 4x)\ dx$$

a) Evaluate this definite integral as the limit of an appropriate Riemann sum. You may find the formulas below useful.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad \text{and} \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

b) Evaluate this definite integral using an antiderivative of $y = x^2 + 4x$ and the Evaluation Theorem.

---

9) Find the following indefinite integrals:

a) $\int \left(\sqrt{\tan(x)}\ \sec^2(x)\right)\ dx$

b) $\int \left(\frac{2\sqrt[3]{x} + 8x^4}{2\sqrt{x}}\right)\ dx$

c) $\int (x(3x^2 + 5)^8)\ dx$

d) $\int \left(\frac{2\cos(x)}{\sin^2(x)}\right)\ dx$