

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2.5 Hours

Mathematics 151

Spring 2015

Instructions:

Answer all questions.

Show all work.

1. Compute the following limits. (Both ∞ and $-\infty$ are acceptable answers.)

(a) (4 points) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{x}$

(b) (4 points) $\lim_{x \rightarrow 3^-} \frac{|3 - x|}{3 - x}$

(c) (4 points) $\lim_{x \rightarrow -2^-} \frac{x + 2}{x^2 + 4x + 4}$

(d) (4 points) $\lim_{x \rightarrow -\infty} \frac{4x + 1}{\sqrt{9x^2 + 1}}$

2. (8 points) Find the derivative of $f(x) = \sqrt{1 - x^2}$ using only the definition of the derivative.

3. Compute $\frac{dy}{dx}$ for the following functions. (You need not simplify.)

(a) (4 points) $y = 2x^9 + \frac{1}{x^2} + 10x^{2/5} + 2\pi + \csc x$

(b) (4 points) $y = \cos(x^2 + \cos 2x)$

(c) (4 points) $y = (x^3 - 1)^5(2x + 1)^4$

(d) (4 points) $y = \frac{\sqrt{x + 1}}{(x^2 + 1)^2}$

4. (8 points) Find an equation for the tangent line to the curve

$$2 \sin x \cos y = 1 \text{ at the point } \left(\frac{\pi}{4}, \frac{\pi}{4} \right).$$

5. (6 points) A snowball is melting in such a way that its radius is decreasing at a rate of 2 centimeters per hour. At what rate is the snowball's volume changing when the volume is 288π cubic centimeters? (Hint: The volume of a ball of radius r is $V = \frac{4}{3}\pi r^3$.)

6. (12 points) Let $f(x) = \frac{x}{x^2 - 4}$.

Find the intervals of increase/decrease and the intervals of concavity, as well as all local maxima and minima, inflection points, and asymptotes. Use this information to sketch the graph of $f(x)$.

7. (4 points) Find a function $f(x)$ such that $f'(x) = \sin 4x$ and $f(0) = 0$.

8. (8 points) What point on the graph of $y = \frac{1}{x^3}$ for $x > 0$ is the closest to the origin?

9. (6 points) Estimate the area under the graph $y = 4 - x^2$ from $x = -2$ to $x = 2$ using four approximating rectangles and right endpoints.

10. **Without using your calculator**, compute the following integrals.

(a) (4 points) $\int_0^4 (6\sqrt{x} + x^3) dx$

(b) (4 points) $\int_0^4 |3 - x| dx$

(c) (4 points) $\int_{-\pi}^{\pi} x^2 \sin(x^3) dx$

11. (4 points) Let $f(x) = \int_0^{x^2} \cos(u^2) du$. Compute $f'(x)$.