

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION

$2\frac{1}{2}$ HOURS

MATHEMATICS 151

SPRING 2016

INSTRUCTIONS: SHOW ALL WORK IN BLUE BOOKS FOR ALL QUESTIONS.

1. Evaluate the following limits. (Answers could be $\pm\infty$ or DNE)

a) $\lim_{x \rightarrow 4} \frac{x - 4}{|x - 4|}$

b) $\lim_{x \rightarrow +\infty} \frac{x^2 - 3x^3 - 18}{4x^3 - x - 42}$

c) $\lim_{x \rightarrow 0} \frac{\sin^2(3x)}{x^2}$

d) $\lim_{x \rightarrow -3} \frac{\sqrt{x + 12} - 3}{x + 3}$

2. For what values of x is the function $Y(x) = \begin{cases} \frac{x - 1}{\sqrt{x} - 1} & \text{if } x > 1 \\ \frac{5 - 3x}{6} & \text{if } -2 \leq x \leq 1 \\ \frac{6}{x - 4} & \text{if } x < -2 \end{cases}$ discontinuous?

3. Let $f(x) = \sqrt{4x + 1}$. Using only **the definition of derivative**, find $f'(x)$.

4. In each of the following, find $\frac{dy}{dx}$. (Algebraic simplification is not needed)

a) $y = \sqrt[3]{x} - x^8 + \frac{5}{x^4} - \pi^5$

b) $y = \frac{x^3 + 2x}{\cos x}$

c) $y = (x^2 - \tan x)^5 (x^5 + 7)^2$

d) $y = \int_{x^2}^9 \sqrt{16 + t^2} dt$

e) $\cos(x + y) = y \sin(x^3)$

5. a) Find an equation for the tangent line to the graph of the function $H(x) = x^4 + 3x$ at the point $(2, 22)$.
b) Use a linear approximation (differentials) to estimate $H(2.001)$.

(CONTINUED ON THE BACK)

6. Two cars start moving from the same point. One car travels east at 60 mph and the other travels north at 80 mph. At what rate is the distance between the cars increasing five hours later?
7. Given $g(x) = \frac{x^2}{x^2 - 1}$.
- Find g' and g'' and simplify both.
 - Find all critical points of g .
 - Find the intervals on which g is increasing and those on which it is decreasing.
 - Find all local extrema of g .
 - Find the intervals where g is concave upward, where it is concave downward, and any inflection points of g .
 - Find any horizontal or vertical asymptotes of g .
 - Sketch the graph of g . (The graph should agree with your answers from parts a) to f.)
8. Find two nonnegative numbers x and y whose sum is 300 such that x^2y is as large as possible.
9. Find the following indefinite integrals:
- $\int \frac{\sqrt{x} - 3}{\sqrt{x}} dx$
 - $\int [\sin(5x) + 7]^4 \cos(5x) dx$
10. Evaluate the following definite integrals:
- $\int_0^\pi \cos\left(3x - \frac{\pi}{2}\right) dx$
 - $\int_{-1}^1 x^3(x^4 - 5)^6 dx$
11. Evaluate $\int_0^2 (3x^2 + 1) dx$ as a limit of a Riemann sum. (Hint: $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$.)