QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 151 **Spring 2017**

Instructions: Show all work in your blue book for all questions.

1. Evaluate each of the following limits. (Answers could be $\pm\infty$ or DNE.)

   a) $\lim\limits_{x\to+\infty} \dfrac{32-6x^3+2x^2}{-3x^3-9x^2-16}$

   b) $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^3-2x^2+x-2}$

   c) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$

   d) $\lim\limits_{h\to 0} \dfrac{(x+h)^3-x^3}{h}$

   e) $\lim\limits_{x\to 2^-} \dfrac{x^2-2x}{x^2-4x+4}$

2. For what value(s) of x is the function

$$y(x)=\begin{cases} 5 & \text{if } x<-3 \\ 2x & \text{if } -3\le x<1 \\ x^2+1 & \text{if } x\ge 1 \end{cases}$$ discontinuous?

3. In each of the following, find $\frac{dy}{dx}$. (Algebraic simplification not needed.)

   a) $y=\dfrac{x^2-x-\pi^2}{\sin x}$

   b) $y=\sqrt[5]{x^2+1}-7x^6+\dfrac{2}{x^3}$

   c) $y=(x^2+\cos x)^4(-x^3+7x)^5$

   d) $y=\displaystyle\int_{3x^2}^{10}\sqrt{t^2+2}\,dt$

   e) $2xy-3x^2y^2-6=0$

4. a) Find an equation for the tangent line to the graph of the function $g(x)=4x^3-2x$ at the point $P(1,2)$.

   b) Use a linear approximation (differentials) to estimate $g(1.0002)$.

5. The altitude of a triangle is increasing at the rate of 2 cm/min. while the area of the triangle is increasing at 4 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2 ?

(continued on the back)

6. Find the point on the line $y = 4x - 1$ that is closest to the point $P(3,0)$.

7. Let $f(x) = \dfrac{x^2}{x^2+4}$.

   a) Determine any vertical or horizontal asymptotes of the graph of f.
   b) Find f', f'' and all critical numbers of f.
   c) Identify intervals where f is either increasing or decreasing and find any local extrema.
   d) Identify intervals where f is either concave up or concave down and find inflection points, if any exist.
   e) Use the information found in parts a) – d) to sketch the graph of f. Your graph should make sense based upon your results for a) – d).

8. Evaluate the following integrals:

   a) $\displaystyle\int \sin^3(2x)\cos(2x)\,dx$

   b) $\displaystyle\int \frac{\sqrt[3]{x} - 5x^3 + 2.5x^{6.5}}{x}\,dx$

   c) $\displaystyle\int x^4 \tan(x^5)\sec^2(x^5)\,dx$

9. Compute $\displaystyle\int_0^2 (3x^2 + 2x + 1)\,dx$ by using the limit of an appropriate Riemann sum.

   ($\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$; $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$)

10. a) Find the average value of $f(x) = 3x^2 + 2x + 1$ on the interval $[0, 5]$ using the Mean Value Theorem for integrals.

    b) Use the Intermediate Value Theorem to show that there is a root of the equation $5x^3 - 7x^2 + 4x - 3 = 0$ in the interval $(-1, 2)$. Then use your calculator to estimate the root, accurate to three decimal places.