QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

MATHEMATICS 151 **SPRING 2018**
<u>INSTRUCTIONS</u>: SHOW ALL WORK IN YOUR BLUE BOOK FOR ALL QUESTIONS.

1. Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$ or does not exist, explain why.

a) $\lim_{x\to -5} \dfrac{x+5}{x^2+7x+10}$

b) $\lim_{x\to 4^+} \dfrac{\sqrt{x+5}-3}{x-4}$

c) $\lim_{t\to 0} \dfrac{t\cos(2t)}{\sin(5t)}$

d) $\lim_{x\to \infty} \dfrac{7x-8}{\sqrt{9x^2+10}}$

e) $\lim_{x\to 3^-} \dfrac{x^2-8x+15}{x^2-6x+9}$

2. a) State the definition that a function $f(x)$ is continuous at $x = a$.
b) Is there a value of k which will make f continuous at $x = 9$? If yes, what is it and why? If no, explain why not.

$$f(x) = \begin{cases} \dfrac{\frac{1}{x-5}-\frac{1}{4}}{x-9} & x \neq 9 \\ k & x = 9 \end{cases}$$

3. a) Use the <u>definition</u> of the derivative to find $f'(x)$ if $f(x) = \sqrt{3x-2}$.
b) Write an equation of the tangent line to the curve $y = \sqrt{3x-2}$ at $x = 6$.

4. Find the following derivatives. (You need not simplify.)

a) $y = (\cos 4x - 5x^{-3})^2 \sqrt[3]{2x^2+7}$

b) $y = \dfrac{x^{3/2}-5}{1+\sin^2(3x)}$

c) $y^3 = \tan(x+y)$

d) $g(x) = \int_4^{x^2} \sqrt{1+t^3}\, dt$

5. Use linear approximation (i.e, differentials) to estimate the value of $\frac{1}{2.08}$.

(CONTINUED ON THE BACK)

6. Let $f(x) = \frac{x+2}{x^3} \left(= \frac{1}{x^2} + \frac{2}{x^3}\right)$.

a) Find $f'(x)$ and $f''(x)$ and simplify both.
b) Find intervals on which f is increasing and those on which f is decreasing.
c) Find intervals on which f is concave upward and those on which f is concave downward.
d) Find any horizontal or vertical asymptotes of the graph of f.
e) Using the information found in parts a) – d), sketch the graph of f. Label intercepts, local extrema and inflection points, if any.

7. Find each of the following integrals:

a) $\int \frac{5x^2 - 6x + 3}{\sqrt{x}} dx$

b) $\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx$

c) $\int x\sqrt{2-x}\, dx$

8. Sand falls onto a conical pile at the rate of 10 cubic feet per minute. The radius of the base of the pile is always equal to one-half of its altitude. How fast is the altitude of the pile increasing when the pile is 15 feet deep? [Note: $V_{cone} = \frac{1}{3}\pi r^2 h$]

9. Find the coordinates of the point on the curve $y = \sqrt{x}$ which is nearest to the point $(2, 0)$.

10. a) Use the limit of a Riemann sum to find the area of the region bounded by the graph of $f(x) = 9 - x^2$, the x-axis, and the vertical lines $x = 0$ and $x = 1$.

[Note: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$]

b) Use an appropriate definite integral to compute the area of the region described in part a).