QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 151 **Spring 2019**

Instructions: Answer all the questions. Show all work.

1. Use analytical methods (not your calculator or L'Hôpital's Rule) to find each of the following limits, allowing for $+\infty$ or $-\infty$ as possible answers.

   a) $\lim\limits_{x\to 5} \dfrac{\sqrt{9-x}-2}{x-5}$

   b) $\lim\limits_{x\to -3} \dfrac{9-x^2}{x^2+8x+15}$

   c) $\lim\limits_{x\to +\infty} \left(\sqrt{x^2+3x}-x\right)$

   d) $\lim\limits_{x\to -2^-} \dfrac{5x+3}{x^2+7x+10}$

   e) $\lim\limits_{x\to 0} \dfrac{\tan^2 2x}{\sin^2 3x}$

2. Let $f(x) = \begin{cases} \dfrac{x^2+x-12}{x^2-9} & x \neq 3 \\ K & x = 3 \end{cases}$

   Use the definition of continuity to determine whether there is a value of K which will make f continuous at $x = 3$. If there is a value of K, what is it and why? If there is no value, explain why not.

3. a) Use the definition of the derivative to find $f'(x)$ if $f(x) = \frac{4}{x^2}$.

   b) Write an equation of the tangent line to the graph of $y = \frac{4}{x^2}$ at the point $(2, 1)$.

4. In each of the following find dy/dx. (You need not simplify.)

   a) $y = \left(6\sqrt[3]{x} + \dfrac{3}{x^5}\right)^4 (1+\sin^2 x)^7$

   b) $y = \dfrac{\pi^3 + \tan^5(x^4)}{4x^{3/4} - x}$

   c) $y = x\sec^3 y + 2x^3y^4$

   d) $y = \displaystyle\int_6^{\sin x} \sqrt[3]{t^4+5}\, dt$

(continued on the back)

5. a) Use the Intermediate Value Theorem to show that $x^5 + x^3 + x - 7 = 0$ has a positive root.

b) Use Rolle's Theorem to show that $x^5 + x^3 + x - 7 = 0$ has exactly one root.

6. Let $f(x) = \dfrac{x^2 - 1}{x^2 + 4}$.

a) Find $f'(x)$ and $f''(x)$ and simplify both.
b) Find intervals on which f is increasing and those on which f is decreasing.
c) Find intervals on which f is concave up and those on which f is concave down.
d) Find any horizontal or vertical asymptotes of the graph of f.
e) Using the information found in parts a) to d), sketch the graph of f. Label intercepts, local extrema and inflection points, if any.

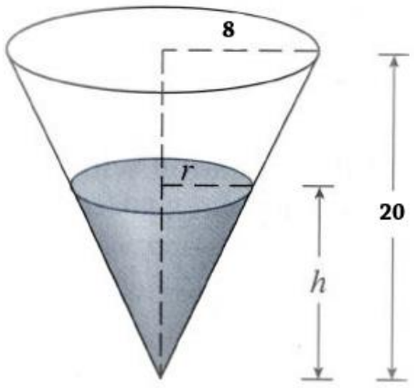
7. Find each of the following:

a) $\displaystyle\int_1^8 \left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}}\right) dx$

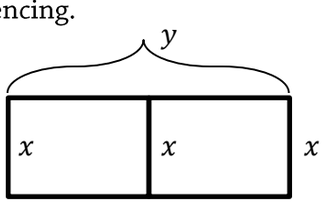
b) $\displaystyle\int \frac{x + \cos 2x}{\sqrt{x^2 + \sin 2x}}\, dx$

c) $\displaystyle\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}}\, dx$

8. An inverted conical tank has a radius of 8 feet at the top and is 20 feet high (see diagram). If water flows into the tank at the rate of $15\ ft^3/min$, how fast is the water level rising when the water is 10 feet deep?
[Note: $V_{cone} = \frac{1}{3}\pi r^2 h$]



9. A rectangular field, having an area of 1200 square feet, is to be enclosed by a fence costing \$3 per running foot. An additional fence which is to be used to divide the field down the middle costs \$2 per running foot (see diagram). Find the dimensions of the field that will minimize the total cost of the fencing.



10. a) Use the limit of a Riemann sum to find the area of the region bounded by the graph of $f(x) = 7 - 3x$, the x-axis, and the vertical lines $x = 0$ and $x = 2$. [Note: $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$]

b) Use an appropriate definite integral to compute the area of the region described in part a)