QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 151 **Spring 2022**

<u>Instructions:</u> <u>Answer all questions.</u> <u>Show all work.</u>

1) Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$ or does not exist, explain why.

a) $$\lim_{x \to 3} \frac{x^2 - 10x + 21}{x^2 - 9}$$

b) $$\lim_{x \to 5^+} \frac{\sqrt{2x-1} - 3}{x - 5}$$

c) $$\lim_{\theta \to 0} \frac{\sin 6\theta}{3\theta + \tan 4\theta}$$

d) $$\lim_{x \to 4^-} \frac{x^2 - 2x - 8}{|x - 4|}$$

e) $$\lim_{x \to -\infty} \frac{6x^4 + 7x^2 + 1}{5x^3 + 8x}$$

2) Let $f(x) = \sqrt{5 - x}$. <u>Using the definition of the derivative</u>, find $f'(x)$.

3) In each of the following find $\frac{dy}{dx}$. (You need not simplify.)

a) $y = \sqrt[4]{3x^3 + 5}(\sin 6x - 2x^4)^3$

b) $$y = \frac{\sec 4x}{1 + \tan 6x}$$

c) $y = \sin(\cos \pi x)$

d) $\tan\left(\frac{2x}{y}\right) = 4x^2 + 4y$

e) $$y = \int_{\sin x}^{4} \sqrt{2 + t^3}\, dt$$

4) Show that the equation $3x + \cos x - 2 = 0$ has exactly one real root. Justify your conclusion by using appropriate theorems.

5) Use a linear approximation (i.e., differentials) to estimate $(15.97)^{\frac{3}{4}}$.

(continued on the back)

6) Let $f(x) = \dfrac{3x^2}{x^2 - 4}$.

a) For which intervals is f increasing and for which is f decreasing?
b) Find all local maxima and/or local minima of f.
c) Find any and all vertical and horizontal asymptotes of the graph of f.
d) For which intervals is the graph of f concave up and for which is it concave down?
e) Find the inflection point(s) of the graph of f, if any.
f) Sketch the graph of $y = f(x)$ using the information found in parts a) – e).

7) If a snowball melts so that its surface area decreases at a rate of $2\ cm^2/min$, find the rate at which the diameter decreases when the diameter is $8\ cm$.
(Note: Surface area of a sphere: $S = 4\pi r^2$)

8) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 12 - x^2$. Make sure to justify that your answer gives a maximum value.

9) Evaluate $\int_0^2 (x^2 + 3x)dx$ as the limit of a Riemann sum.

$$\left(\text{Note: } \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad , \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \right)$$

10) Find each of the following integrals.

a) $\displaystyle\int \frac{4x^3 - 2x + 3}{\sqrt{x}}\, dx$

b) $\displaystyle\int \frac{\cos\left(\frac{1}{x}\right)}{x^2}\, dx$

c) $\displaystyle\int x\sqrt{1-x}\; dx$