

**Queens College
Department of Mathematics**

Final Examination

$2\frac{1}{2}$ Hours

Mathematics 152

Fall 2017

Instructions: Show all work.

1. In each of the following, find $f'(x)$, writing your answer in simplest form. State the domains of $f(x)$ and $f'(x)$.
 - a) $f(x) = \log_2(\arctan x)$
 - b) $f(x) = \ln(\ln x^2)$
 - c) $f(x) = 2^{\ln x}$
 - d) $f(x) = \sin^{-1} \frac{1}{x}$

2. Without using your calculator, find the following limits. Give explanations for your answers.
 - a) $\lim_{x \rightarrow -\infty} \tan^{-1}(\ln x^{-2})$
 - b) $\lim_{x \rightarrow 3^-} 3^{\left(\frac{x^2+4}{x-3}\right)}$
 - c) $\lim_{x \rightarrow 0} \frac{\sin \pi x}{\sin 7x}$
 - d) $\lim_{x \rightarrow \infty} e^{-x^2} \sqrt[3]{1+x^2}$

3. Find each of the following indefinite integrals:
 - a) $\int \tan^6 x \, dx$
 - b) $\int \sin^3 x \sin 2x \, dx$
 - c) $\int e^{-3x} \sin 3x \, dx$
 - d) $\int \frac{dx}{x^2 \sqrt{x^2 - 16}}$

4. Without using your calculator, evaluate the given definite integrals. If an improper integral is divergent, state this as your answer.
 - a) $\int_0^1 \frac{dx}{(x^2 + 1)^2}$
 - b) $\int_9^{25} \frac{\ln x}{x^3} \, dx$
 - c) $\int_0^1 \sqrt{x} \ln x \, dx$
 - d) $\int_0^\infty x e^{-x} \, dx$

5.
 - a) Set up, but do not evaluate, a definite integral that can be used to compute the arc length of $f(x) = \tan^{-1} x$ when $1 \leq x \leq 3$.
 - b) Estimate the above integral using the Trapezoidal Rule with $n = 6$.

(continued on the back)

6. Find **all** real numbers c such that the area under the graph of $f(x) = \frac{1}{1+x^c}$ and above the interval $[1, \infty)$ on the x -axis is infinite. (Hint: Use the Comparison Test.)
7. a) Find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^3$ and $y = x^2$ around the line $y = -1$
- b) Find the volume of the solid of revolution obtained by rotating the region bounded by $y = (x - 3)^2$ and $y = 4$ around the line $x = 1$.
8. Decide whether each of the following series converges absolutely, converges conditionally or diverges. Clearly specify the criteria that you are using.
- a)
$$\sum_{n=1}^{\infty} n^{-e}$$
- b)
$$\sum_{n=1}^{\infty} (-e)^n$$
- c)
$$\sum_{n=1}^{\infty} \frac{1}{n \ln(n^2 + 1)}$$
- d)
$$\sum_{n=1}^{\infty} \left(\frac{3n + 7}{7n - 3}\right)^{3n}$$
9. Find the interval and radius of convergence of the power series
- $$\sum_{n=1}^{\infty} (-1)^n \frac{(x - 2)^n}{n^3 4^n}$$
10. a) Find $T_2(x)$, the second Maclaurin polynomial of $f(x) = e^{-x^2}$.
- b) Compute $T_2(0.1)$ and use Taylor's Theorem to find the largest possible error that can result when $T_2(0.1)$ is used to estimate $f(0.1)$.
11. *(extra credit) Prove that if $\lim_{n \rightarrow \infty} a_n = 0$ and a sequence $\{b_n\}$ is bounded, then $\lim_{n \rightarrow \infty} a_n b_n = 0$.