QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 152 **Fall 2018**

<u>Instructions</u>: Answer all questions. Show all work.

1. Differentiate each of the following functions: (algebraic simplification is unnecessary)

 a. $y = \tan^{-1}(6x+1) + \ln(\sec x)$

 b. $y = x^{9\cos x}$

 c. $f(t) = e^{t\sin(2t)} - 3\sin^{-1}(\sqrt{t})$

2.
 a. Evaluate each of the following indefinite integrals:

  i. $\displaystyle\int \frac{3x^2+x}{(x-1)(x+1)^2}\,dx$

  ii. $\displaystyle\int x\cdot 7^x\,dx$

 b. Without using your calculator, find the exact value of each of the following definite integrals.

  i. $\displaystyle\int_0^{\frac{\sqrt{3}}{5}} \frac{1}{1+25x^2}\,dx$

  ii. $\displaystyle\int_0^1 2x^3\sqrt{1-x^2}\,dx$

3. Determine whether the integral below is convergent or divergent. If it is convergent, find its value.

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$$

4. Find the solution of the differential equation below that satisfies the given initial condition.

$$\frac{dy}{dt} = \frac{2t + sec^2(t)}{2y}, \quad y(0) = -6$$

5. Let R be the region in the plane bounded by the graphs of $y = x^2 + 2x$ and $y = 2 + x$.

 a. Find the area of R.

 b. Find the volume of the solid of revolution generated by revolving R around the line $y = 3$.

 c. Set up, <u>but do not evaluate</u>, an expression that computes the perimeter of R.

(continued on the back)

6. Discuss the convergence or divergence of each of the following series. Explain each of your answers, stating the names of the tests you are using.

a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$

b. $\sum_{n=1}^{\infty} \frac{n^2}{6n^4 - 5}$

c. $\sum_{n=1}^{\infty} \frac{(-17)^n}{n!}$

d. $\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n+1}\right)^n$

7. Find the radius of convergence and interval of convergence of the power series below.

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{\sqrt{n} \cdot 4^n}$$

8. Compute each of the following limits:

a. $\lim_{x \to 2} \frac{\sin(\pi x)}{\ln(x-1)}$

b. $\lim_{x \to -\infty} \left(1 + \frac{1}{2x}\right)^{3x}$

9. a. Starting with the Maclaurin series for e^x, write the Maclaurin series for e^{-x^2}.

b. Using the first four terms of the series found in part (a), obtain an estimate for $\int_0^1 e^{-x^2}\,dx$.

10. a. Find $T_3(x)$, the third Taylor polynomial of $f(x) = \sqrt{x}$ centered at $a = 1$.

b. If $T_3(x)$ is used to estimate $f(x)$ for $x \in [.9, 1.1]$, use $|R_3(x)|$ to find the largest possible error that can result.