QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAM
2 ½ hours

MATH 152 **FALL 2019**

Show all work. Unless otherwise specified, all integrals must be evaluated using calculus (i.e. not your calculator).

1. (a) Compute the numerical value of $e^{2\ln 3}$.

   (b) Find an equation of the line tangent to $y = \dfrac{(x^3+2)\sqrt{x^2+1}}{(2x+1)^3}$ at the point where $x = 0$.

   (c) If $f(x) = \ln(2x+3)$ find $f^{-1}(0)$ and $\left(f^{-1}\right)'(0)$.

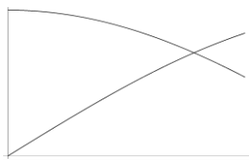
2. Evaluate each of the following limits:

   (a) $\lim\limits_{x\to 0} \dfrac{xe^x - \sin x}{\sin(x^2)}$ (b) $\lim\limits_{x\to 0} (1+x\ln 2)^{\frac{1}{x}}$

3. Let R be the region in the first quadrant bounded by $y = \cos x$, $y = \sin x$ and the y-axis.

   (a) Compute the area of R.

   (b) If S is the solid of revolution obtained by rotating R about the x-axis, compute the volume of S.

   (c) If T is the solid of revolution obtained by rotating R about the y-axis, compute the volume of T.

4. Let L be the length of arc of the curve $y = \sin x$, $0 \le x \le \pi$. Use your calculator to approximate L to 4 decimal places.

5. Evaluate using techniques of integration. Calculator approximations will not be accepted.:

   (a) $\int_0^\infty x^2 e^{-x^3}\,dx$ (b) $\int \dfrac{3x^2+x+1}{x^3+x}\,dx$ (c) $\int_0^1 x^3\sqrt{1-x^2}\,dx$

6. Use an appropriate test to determine if the series converges or diverges. Show all details *clearly* indicating which test you are using.

   (a) $\sum\limits_{n=1}^{\infty} \dfrac{n(n+1)}{(n+2)(n+3)}$ (b) $\sum\limits_{n=0}^{\infty} \dfrac{2^n}{3^n+4^n}$ (c) $\dfrac{1}{e} + \dfrac{2}{e^2} + \dfrac{3}{e^3} + \dfrac{4}{e^4} + \ldots$

(continued on the other side)

7. Determine if each of the following series converges absolutely, converges conditionally or diverges. Show all details, *clearly* indicating which tests you are using.

(a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{2^n}$ (b) $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\ldots$

8. Let $f(x)=\sum_{n=0}^{\infty}\frac{1}{(n+1)3^n}x^{n+1}=x+\frac{x^2}{6}+\frac{x^3}{27}+\frac{x^4}{108}+\ldots$

(a) Find the interval of convergence of this series. (Be sure to check the endpoints.)

(b) Find the function which represents $f'(x)$ within its interval of convergence.

9. (a) Find the Maclaurin series representing $f(x)=x^3\sin(x^2)$.

(b) Use the result of (a) and the alternating series approximation theorem to approximate $\int_0^1 f(x)\,dx$ with 4 decimal place accuracy.