QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 152 Spring 2015

Instructions. Answer each question. Show your work and justify your answers. Partial credit will be awarded for relevant work.

1. The domain of a function f is the interval $(0, 5)$ and its range is the interval $(-3, 12)$. The derivative $f'(x)$ is positive for all x in $(0, 5)$. Here are some values of f and f':

x	1	2	3	4
$f(x)$	2	5.5	8	9.5
$f'(x)$	4	3	2	1

a. Justify that there is an inverse function $g = f^{-1}$. What is domain and range of g?

b. Find the numerical value of i) $g(2)$ and ii) $g'(2)$.

2. Find the derivative $\frac{dy}{dx}$ for each of the following. Please make basic simplifications.

a. $y = (e^{2x} + 1)^x$ b. $y = e^{3x} \tan^{-1}(4x)$

3. Find each of the following indefinite integrals.

a. $\displaystyle\int x^5 \ln x \, dx$ b. $\displaystyle\int \frac{1}{\sqrt{16 - 9x^2}} \, dx$ c. $\displaystyle\int \frac{x+9}{(x+1)(x+3)^2} \, dx$

4a. Let R be the region enclosed by the graphs of $y = 8x - x^2$ and $y = 2x$. Set up definite integrals to represent each of the following, but do not antidifferentiate.

i) the area of R;

ii) the volume of the solid obtained by revolving R around the x-axis;

iii) the volume of the solid obtained by revolving R around the line $x = 10$.

b. Set up a definite integral to represent the arc length of the sine curve $y = \sin x$ in the first quadrant $0 \le x \le \pi/2$ and approximate the integral to two decimal places. You may use suitable calculator functions.

5. Suppose that $y = f(x)$ is differentiable for all $x > 0$ and the slope of the tangent to the graph of $f(x)$ at each point $x > 0$ satisfies

$$\frac{dy}{dx} = \frac{(x+2)y}{x}.$$

If the graph passes through the point $(1, 2)$, find y in terms of x.

(continued on other side)

6. For both parts of this problem, let $a_n = \dfrac{\ln(n+4)}{\ln(n^2+9)}$.

a. Evaluate $\lim_{n \to \infty} a_n$.

b. Does the series $\sum_{n=1}^{\infty} a_n$ converge? Justify your answer.

7. Determine whether or not each of the following series converges. In each case, state the test you are using and explain briefly why it applies.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n^2+9)}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n + 2\cos n}$

8a. Find the center and the radius of convergence of the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{n}{3^n}(x-2)^n = \frac{1}{3}(x-2) + \frac{2}{9}(x-2)^2 + \frac{3}{27}(x-2)^3 + \dots$$

b. Find the power series for the derivative $g(x) = f'(x)$ having the same center as in part a. What is the radius of convergence of the power series for $g(x)$?

9. Let $T_2(x)$ be the degree 2 Maclaurin polynomial (i.e. Taylor polynomial with center at 0) for the function $F(x) = \sqrt{9+4x}$.

a. Find $T_2(x)$.

b. State the form of the remainder term $R_2(x)$ according to Taylor's theorem.

c. Find an explicit real number k such that $0 \le F(x) - T_2(x) \le kx^3$ for all $x \ge 0$.

Suggestion: you may use the fact that $F^{(3)}(x) = \frac{24}{(9+4x)^{5/2}}$.