# Queens College
# Department of Mathematics

## Final Examination
## 2.5 Hours

**Mathematics 152** **Spring 2016**

**INSTRUCTIONS:** YOU MUST SHOW ALL DETAILS OF YOUR ARGUMENTS TO RECEIVE CREDIT.

1. Find the volume of the solid of revolution formed by revolving the region between the curve $y = \sqrt{x}\exp(-x)$ and the $x$-axis about the $x$-axis for $x$ in the interval $(1, \infty)$.

2. Show that, for all $x$, $-1 < x < 1$,
$$\frac{d \arccos x}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

3. Calculate the following integrals:

   (a) $$\int \exp(x)\sin(x)dx \quad \text{(use integration by parts)}$$

   (b) $$\int_{-1}^{1} x\exp(x^2)dx$$

   (c) $$\int_{-\infty}^{+\infty} \frac{1}{1+x^2}dx$$

4. Write the partial fraction decomposition of
$$\frac{x+1}{(x^2+1)^3}.$$

5. Calculate the length of the curve given by the equation $y^2 = x^3$ between the points (0,0) and (1,1).

6. Find a differentiable function $y(x)$ defined on $(0, +\infty)$ such that
$$\frac{dy}{dx} = \frac{y}{2x} \quad \text{and} \quad y(4) = 6.$$

7. Calculate
$$\lim_{n \to +\infty} \frac{n!}{n^n}.$$

8. Show that the series
$$\sum_{n=1}^{+\infty} \frac{1}{n^{152}}$$
converges using the integral test.

9. Determine the radius of convergence of the power series
$$\sum_{n=2}^{+\infty} \frac{x^n}{(n-1)!}.$$

10. Find a power series representation of $\ln(x+1)$.

11. Find the Maclaurin series (Taylor series about $x = 0$) of $x^{1776} + x^7 + x^4$.

12. Show that if the series $\sum_{n=1}^{+\infty} a_n$ is convergent, then $\lim_{n \to +\infty} a_n = 0$.

13. For all $x > 0$, define
$$L(x) = \int_1^x \frac{1}{t}dt.$$
Show that, for all $x, y > 0$,

    (a) $$L(xy) = L(x) + L(y).$$

    Hint: for each $y > 0$, consider the function $f(x) = L(xy)$ (where $x > 0$), calculate the derivatives $\frac{df}{dx}$ and $\frac{dL}{dx}$, compare the results, consider formula (a) substituting $x = 1$, and make the conclusion.