QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 152 **Spring 2017**

<u>Instructions</u>: Answer <u>all</u> questions. <u>Show</u> <u>all</u> <u>work</u>.

1. Let R be the region in the plane bounded by the graphs of $y = kx^2$ and $y = kx$, where k is a positive constant.
   (a) Find the volume of the solid of revolution obtained when R is rotated about the x-axis. (Answer will be in terms of k.)

   (b) Find the volume of the solid of revolution obtained when R is rotated about the y-axis. (Answer will be in terms of k.)

   (c) For what value of k will the solids in parts (a) and (b) have the same volume?

2. Let R be the region in the plane bounded by the graph of $y = e^{x^2}$ and the lines $x = 1$ and $y = 1$.
   (a) USE YOUR CALCULATOR to find the area of R. Round your answer to the nearest hundredth.

   (b) USE YOUR CALCULATOR to find the perimeter of R. Round your answer to the nearest hundredth.

3. Let $y = x^{\left(\frac{\ln 2}{1+\ln x}\right)}$
   (a) Find $\frac{dy}{dx}$

   (b) Find $\lim_{x\to 0^+} y$

4. Integrate:
   (a) $\int \frac{x^2 + 3x - 2}{(x-1)(x^2-1)}\,dx$

   (b) $\int \frac{\sqrt{x^2-1}}{x^3}\,dx$

   (c) $\int \frac{\cot^3(\sqrt{x})\csc^3(\sqrt{x})}{\sqrt{x}}\,dx$

   (d) $\int_0^e x^2 \ln x\,dx$, if it converges

5. The population of fruit flies in a container grows exponentially and thus obeys the law of exponential growth. If the original population doubles in 3 days, how long will it take for the population to be one hundred times its original size? Round your answer to the nearest hundredth.

(continued on the back)

6. Indicate whether each of the following statements is " TRUE " or " FALSE " :

(a) A bounded sequence is convergent.

(b) If $\sum_{k=1}^{+\infty} a_k$ diverges and $\sum_{k=1}^{+\infty} b_k$ diverges, then $\sum_{k=1}^{+\infty} (a_k + b_k)$ diverges.

(c) The sum of the series $\sum_{k=0}^{+\infty} \frac{(-1)^k}{(2k+1)!}\left(\frac{\pi}{2}\right)^{2k+1}$ is equal to 1.

(d) If the nth term of an infinite series approaches zero as $n \to +\infty$, then the series is convergent.

(e) An infinite series of positive terms converges if and only if its associated sequence of partial sums is bounded above.

7. Determine the convergence or divergence of each of the following series. Give reasons for your conclusions.

(a) $\sum_{k=1}^{+\infty} \frac{(k!)^3}{(3k)!}$

(b) $\sum_{k=1}^{+\infty} \frac{\cos(k^2)}{k^2+1}$

(c) $\sum_{k=1}^{+\infty} \cos\left(\frac{k^2}{k^2+1}\right)$

(d) $\sum_{k=1}^{+\infty} \frac{3^k - 2^k}{6^k}$

8. Determine the interval of convergence of the power series $\sum_{k=0}^{+\infty} \frac{(2x+1)^k}{(k+1)\,3^{k+1}}$. Classify any convergence as either absolute or conditional.

9. Let $f(x) = \ln(2x - 1)$.

(a) Find $T_3(x)$, the third Taylor polynomial of f at $a = 1$.

(b) Use Taylor's inequality to bound $R_3(x)$, the third remainder of f. Then use your answer to determine the largest possible error that can result when $T_3\left(\frac{5}{4}\right)$ is used to approximate $f\left(\frac{5}{4}\right)$.

10. (a) Beginning with the Maclaurin series for e^x, write the Maclaurin series for $x^2e^{-x^2}$.

(b) Use the result of part (a) to obtain a series representation for $\int_0^{1/2} x^2 e^{-x^2}\,dx$. Then use the fewest number of terms of the series tot estimate the value of this integral with an error of less than 0.0001.