QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 152 **Spring 2018**

Instructions: Answer all questions. Show all work.

1. $f(x) = x^3 + 2x - 1$.
   (a) Using an appropriate calculation, show that f has an inverse.
   (b) Find $(f^{-1})'(2)$.

2. (a) Find the area of the region enclosed by the curves $f(x) = (x-1)^3$ and $g(x) = x - 1$.
   (b) Find the volume of the solid obtained by rotating the region enclosed by the curves $y = 3x - x^2$ and $y = x$
   (i) about the x-axis
   (ii) about the y-axis
   (iii) about the line $x = 2$.

3. Differentiate: (Algebraic simplification unnecessary.)
   (a) $y = \tan^{-1}(\ln x)$
   (b) $y = 2^{\sin^{-1} x}$
   (c) $y = \sqrt{x e^{\cos x}}$

4. Find the given limits. Show all work.
   (a) $\displaystyle \lim_{x\to 0} \frac{e^x - (1+x)}{x^2}$
   (b) $\displaystyle \lim_{x\to\infty} \left(1 + \frac{1}{3x}\right)^{3x}$

5. Integrate:
   (a) $\displaystyle \int \frac{\ln 2x}{x^2}\, dx$
   (b) $\displaystyle \int \frac{x^2}{\sqrt{25 - x^2}}\, dx$
   (c) $\displaystyle \int \frac{2x^3 - 4x - 8}{(x^2 - x)(x^2 + 4)}\, dx$
   (d) $\displaystyle \int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x}\, dx$

6. Evaluate the improper integral $\displaystyle \int_{-1}^{0} \frac{e^x}{\sqrt{1 - e^{2x}}}\, dx$.

(continued on the back)

7. Find the solution to the initial value problem

$$xydx + e^{-x^2}(y^2 - 1)dy = 0 \quad \text{when } y(0) = 1.$$

8. Tell whether each of the following are absolutely convergent, conditionally convergent or divergent. Explain your reasoning with appropriate tests.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)(n+1)}{\ln(n+1)}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{7^n + 2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 2^n}{n!}$

9. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-5)^n}{n\, 5^n}$.

10. Use a power series to approximate the definite integral $\displaystyle\int_0^1 \cos\sqrt{x}\, dx$ correct to 4 decimal places.

11. (a) Find a third degree Taylor polynomial for $\ln(x-1)$ centered at $a = 2$.
(b) Use (a) to approximate the $\ln(1.2)$.
(c) Use an error theorem to find the largest error that may result when (a) is used to approximate (b).