QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION
2½ HOURS

Mathematics 152 **Spring 2019**

Instructions: **Answer all questions and show all work in the blue book.**

1. Differentiate each of the following functions.

   (a) $f(x) = \sin^{-1}(e^{2x})$ (b) $f(x) = \ln(x^2 \tan^{-1} x)$ (c) $f(x) = (\ln x)^{\cos x}$

2. Find each of the following integrals.

   (a) $\int \sin^{-1} x \, dx$ (b) $\int \frac{1}{\sqrt{9x^2 + 12x}} dx$ (c) $\int \frac{x^2 + 6x + 9}{(x^2 + 9)^2} dx$

3. Find the following limit.

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$$

4. Determine whether the following improper integral converges or diverges. If it converges, find its value.

$$\int_0^3 \frac{x}{x - 1} dx$$

5. Let *R* be the region bounded by two curves $x = y^2 + 2y$ and $x = 3y + 6$.
   [Note: In this problem, you may **use your calculator** to evaluate the integrals.]
   (a) Find the perimeter of region *R*.
   (b) Find the volume of the solid obtained by rotating *R* around the line $y = 3$.
   (c) Find the volume of the solid obtained by rotating *R* around the line $x = -1$.

6. Find the solution to the following differential equation with the given initial condition.

$$y' - 2xy = 4x \quad \text{and} \quad y(0) = -3$$

7. A sample of a chemical with an original mass of 40g decayed exponentially. Assume it decayed to 15g in 60 days. How long would it take the sample to decay to 5g?

8. For each of the following series, determine whether it is absolutely convergent, conditionally convergent or divergent. Justify your conclusions with appropriate tests.

   (a) $\sum_{n=0}^{\infty} \frac{n^2 + \sin n}{n + 4^n}$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2 + 1}}$ (c) $\sum_{n=0}^{\infty} \frac{(2n)!}{n! + e^n}$

9. Use a power series to approximate the following definite integral with four decimal place accuracy:

$$\int_0^{0.6} x^2 \ln(1 + x^2) \, dx$$

10. Let $f(x) = \sin x$.
    (a) Find $T_4(x)$, the fourth Taylor polynomial of *f* centered at $a = \pi/6$.
    (b) Use $T_4(x)$ to approximate $\sin(40°)$ correct to six decimal places.
    (Hint: $40° = 2\pi/9$ radians)
    (c) Use Taylor's Theorem to estimate the largest possible error when $T_4(x)$ is used to approximate $\sin(40°)$.