**QUEENS COLLEGE**
**DEPARTMENT OF MATHEMATICS**
**Final Examination**
**$2\frac{1}{2}$ Hours**

**Mathematics 152** **SPRING 2022**

**<u>Instructions</u>: <u>Answer all questions.</u> <u>Show all work.</u>**

1. Let $f(x) = e^{\frac{\ln(x)}{x^2+1}}$.

   (a) Find $f'(x)$.

   (b) Evaluate $\lim_{x\to\infty} f(x)$.

   (c) Show that $f(x)$ has an inverse for $x$ on $[e^{-0.1}, e^{0.1}]$ and find $(f^{-1})'(1)$.

2. Let R be the region bounded by two curves $y = 5x$ and $y = 5\sqrt{x}$.

   (a) Find the area of R.

   (b) Find the perimeter of R.

   (c) Find the volume of the solid of revolution obtained by rotating R around $y$-axis.

   (d) Find the volume of the solid of revolution obtained by rotating R around $y = -1$.

3. Differentiate the following functions:

   (a) $f(x) = (\tan^{-1}(25x))^2$

   (b) $f(x) = \dfrac{e^{\sqrt{3x^2+1}}}{\sqrt{\sin x}}$ (use logarithmic differentiation)

4. Evaluate each using techniques of integration:

   (a) $\displaystyle\int_{-\infty}^{0} \frac{1}{5-4x}\,dx$

   (b) $\displaystyle\int 3xe^{3x}\,dx$

   (c) $\displaystyle\int \frac{x^3}{\sqrt{x^2+49}}\,dx$

   (d) $\displaystyle\int_{0}^{1} \frac{x-6}{x^2-6x+8}\,dx$

**(continued on the back)**

5. For the sequence $a_n = \frac{\ln(n)}{n}$

(a) Find $\lim_{n\to\infty} a_n$ (If it does not exist, write DNE, and explain why.)

(b) Does $\sum_{n=1}^{\infty} a_n$ converge? Explain.

6. Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Justify your conclusions.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(b) $\sum_{n=1}^{\infty} \frac{n}{5n+1}$

(c) $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$

(d) $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$

7. Find the radius of convergence, R, and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{8^n \cdot n^4}.$$

8. Find the power series representation for the function $f(x) = \frac{\ln(1+x)}{x}$ about $x = 0$.

9. (a) Using the Maclaurin series for $\cos x$, find the Maclaurin series for $f(x) = \cos(x^2)$.

(b) Using the result of part (a), approximate the definite integral $\int_0^1 \cos(x^2)\,dx$ with four decimal place accuracy.