

**QUEENS COLLEGE  
DEPARTMENT OF MATHEMATICS  
FINAL EXAMINATION  
 $2\frac{1}{2}$  HOURS**

Mathematics 110

Fall 2022

**Instructions: Answer all questions. Show all work. Box final answers. All answers should be rounded to the nearest hundredth. Good Luck!!**

1. The math club is voting on which day they should hold their meetings and their preference rankings are listed below:

Day	Group of 5	Group of 10	Group of 15	Group of 2	Group of 7	Group of 9
Monday	1v	1v	3	2	3	2v
Wednesday	2v	3	1v	1v	2 v	3
Thursday	3	2v	2	3	1v	1v

- a) Which day would be selected using the plurality method?
  - b) Which day would be selected using the plurality with runoff?
  - c) Which day would be selected using Borda's method?
  - d) Which day would win the approval vote?
  - e) Which day is the Condorcet winner, if any?
- 2.
- a) If 4,321 votes are cast in an election that is to be decided by plurality, what is the smallest number of votes a winning candidate can have with a six-candidate race if no ties are allowed?
  - b) Suppose the math majors are voting on their favorite field of study. There are 250 votes to be cast in an election among four top subjects – Algebra, Topology, Geometry and Probability. After the first 210 votes are counted, the tallies are as follows: Algebra received 55 votes, Topology received 40 votes, Geometry received 63 votes, and Probability received 52 votes. What is the minimal number of votes Geometry needs to be assured of a win?
3. A local school district is going to be giving 25 teaching assistants to the elementary schools whose students got the most awards.

School Name	Alfa Elementary	Bailey Grammar	Chester Hill	Daley Primary
# of Awards	96	7	55	89

- Apportion the 25 teaching assistants based on the number of awards earned in each school using
- a) Hamilton's method.
  - b) Lowndes' method.
  - c) Webster's method.

(Continued on the back)

4. A random sample of the grades on an exam are listed:
- |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 56 | 76 | 23 | 84 | 99 | 99 | 86 | 67 | 91 | 85 |
| 96 | 83 | 99 | 79 | 65 | 57 | 85 | 85 | 92 | 93 |
- a) Construct a frequency table and histogram with the first exam interval 20-39.
  - b) Find the sample mean and the sample standard deviation.
  - c) Find the five-number summary and construct a box-and-whisker plot.
5. Eight candidates apply for an internship in the Psych Department. Only three applicants are to be selected.
- a) How many distinct ways can three interns be chosen?
  - b) If the tasks are on a first-come, first-served basis, (that is, the first intern will get first choice, and so on), how many ways can the interns be chosen?
6. An experiment has outcomes 0, 1, 2, 3, and 4 with probabilities as shown
- |        |     |     |    |   |     |
|--------|-----|-----|----|---|-----|
| $p(x)$ | .42 | .05 | .2 | ? | .08 |
| $x$    | 0   | 1   | 2  | 3 | 4   |
- a) Find the missing  $p(x)$ .
  - b) Calculate the mean and standard deviation.
  - c) Calculate the probability of having an outcome of at least 3.
  - d) Construct the probability histogram.
7. In a standard 52-card deck (2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A with four suits (clubs, diamonds, hearts, and spades)), find the probability that
- a) One card is picked, and it is an ace.
  - b) An ace is picked and then without replacement, a face card (J, Q, K) is picked.
  - c) An ace is picked, replaced and then a face card is picked.
  - d) A spade is picked and then, without replacement, another spade is picked.
  - e) A spade is picked, replaced, and then another spade is picked.
8. The student loans of employees at a certain company have an approximately normal distribution with  $\mu = \$150,000$  and  $\sigma = \$30,000$ .
- a) Find the percentage of employees with loans less than \$150,000.
  - b) Find the percentage of employees with a loan between \$180,000 and \$200,000.
  - c) Below what amount is 88.30% of the employees' loans at the company?
  - d) If there are 100,000 employees at the company, how many employees have loans above \$250,000?