Final Examination
$2\frac{1}{2}$ Hours

Mathematics 115   **FALL 2022**

<u>**Instructions**</u>: **Answer all questions and show all your work in the provided blue book. Algebraic solutions and simplifications are required. The use of a cellphone is not allowed.**

1.   Two points, $A(4, -2)$ and $B(-1, 3)$, are given:
     (a)   Find an equation of a line that contains the points A and B.
     (b)   Find an equation of a line parallel to $\overline{AB}$ passing through $(0, 0)$.
     (c)   Find an equation of a line perpendicular to $\overline{AB}$ and passing through the midpoint of A and B.
     (d)   If A and B are the end points of a diameter of a circle, what is the radius of the circle?

2.   If $f(x) = x^2 - 2x - 1$ and $g(x) = x - 3$, evaluate:
     (a) $[g(x)]^2 - f(x)$
     (b) $g(2a + 1)$
     (c) $f(1) - 3g(2)$

3.   Find the quotient and the remainder by using long division:
$$(3x^3 - 2 + x) \div (x - 1)$$

4.   Find all the real solutions of x for the following equations:
     (a) $(x - 2)(x - 3) = x^2$

     (b) $\sqrt{3x + 1} - x = 1$

     (c) $x(3x - 5) = 2$

     (d) $\dfrac{4x}{x + 3} + 1 = \dfrac{x}{x + 2}$

5.   Sketch the graph of $3x - 5y = 15$ and label its intercepts on the graph.

6.   Solve the following system of linear equations:
$$\begin{cases} 5x + 2y = -23 \\ y + 10 = -2x \end{cases}$$

7.   Which of the following is not a function? Explain your reasoning.
     (a) $\{(1, 3), (2, -2), (-3, 4), (1, 2)\}$

     (b) $y = \dfrac{1}{x + 1}$

     (c) $y = x^4 - 4x^2$

(Continued on the back)

8. Find the domain of the following:

    (a) $f(x) = \dfrac{x-3}{(x-1)(x+3)}$

    (b) $y = 3$

    (c) $g(x) = 2x^3 - 3x - 1$

    (d) $h(x) = \dfrac{3}{\sqrt{x-2}}$

9. Factor the following completely:

    (a) $x^3 + 3x^2 - xy^2 - 3y^2$

    (b) $6x^2 - 7x - 20$

    (c) $12a^3b - 24a^2b + 12ab$

10. Simplify the following expressions:

    (a) $\dfrac{x^2 + 2x + 1}{x^2 - 1} \div \dfrac{8x^2 + 8x}{4x^3 - 4x^2}$

    (b) $\dfrac{\sqrt[3]{16x^4y^2}}{\sqrt[3]{2xy^5}}$

    (c) $(2a^{\frac{2}{3}}b^2)^3 (8a^3b^6)^{\frac{1}{3}}$

    (d) $(-125)^{\frac{1}{3}} + (2x^{-3} + 7)^0 - (-\dfrac{1}{3})^{-2}$

11. Rationalize the denominators and simplify:
    $$\dfrac{3}{\sqrt{3} + \sqrt{2}} + \dfrac{6}{\sqrt{2}}$$

12. For the function $f(x) = x^2 - 4x + 3$, find the following:
    (a) The coordinates of x-intercept.
    (b) The coordinates of y-intercept.
    (c) An equation of the axis of symmetry.
    (d) The coordinates of the vertex.
    (e) The domain and range in interval notation.

13. The graph of $y = f(x)$ is given on the right. Use the graph of $y = f(x)$ to find the following:

    (a) Domain of $f(x)$ in interval notation.
    (b) Range of $f(x)$ in interval notation.
    (c) The value of x where $f(x) = -2$.
    (d) $f(0)$.
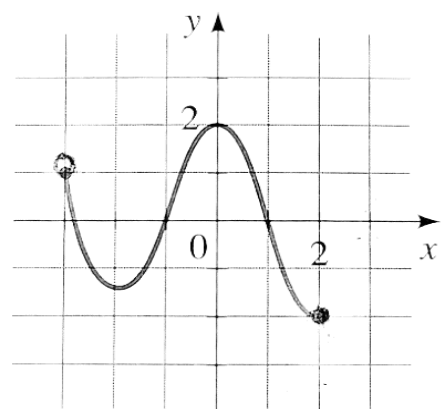
14. How many ounces of pure water should be added to 80 oz. of an 8% salt solution to make a 5% salt solution?