

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION

$2\frac{1}{2}$ Hours

Mathematics 122

Fall 2022

Instructions: Answer all questions. Show all work.

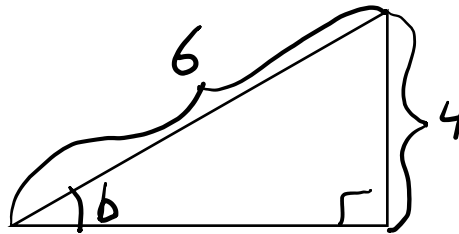
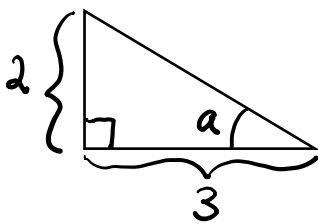
1. Let $F(x) = -\sqrt{x+16}$, $G(x) = -\frac{2}{x}$, and $H(x) = x^2 + 64$.
- Sketch the graph of F and determine its domain and range using interval notation. Label the x - and y - intercepts.
 - Use the graph of F to sketch F^{-1} on the same set of axes. Label both graphs and all intercepts.
 - Find an equation for F^{-1} and determine its domain and range.
 - Find and simplify the difference quotient, $\frac{G(x+h)-G(x)}{h}$, where $h \neq 0$.
 - Find $F \circ H$ and simplify, if possible.
2. Solve each of the following equations for x :
- $\log_2(x^2 + 2x - 15) - \log_2(x - 3) = 4$
 - $5^{x^2-5x} = \frac{1}{25}$ (Express your answer in simplest radical form.)
 - $e^{-2x} = 70$ (Round your answer to three decimal places.)
3. Use appropriate transformations to sketch the graph of each of the following. Label the coordinates of all x - and y - intercepts. Write an equation of any horizontal and vertical asymptotes.
- $y = |x - 3| + 2$
 - $y = -\sqrt[3]{x+4} + 1$
 - $y = 3 - \frac{1}{x+2}$
 - $y = e^{-x} - 4$
 - $y = 2 \cos\left(\frac{x}{3}\right)$, in the interval $[0, 6\pi]$
4. Find the domain of each of the following functions: (Express your answer in interval notation.)
- $f(x) = \frac{21x^5}{7x^2-19x-6}$
 - $g(x) = \frac{\sqrt{2x+7}}{x-5}$
5. Without using your calculator, evaluate the following expressions:
- $\sin(180^\circ) \cos(45^\circ) - \cos(180^\circ) \sin(45^\circ)$
 - $\log_4(8) + \log_4(72) - \log_4(9)$
 - $\tan^{-1}[\sec(3\pi)]$

(Continued on the back)

6. Let $f(x) = -x^2 + 4x + 5$.
- Express $f(x)$ in standard form.
 - Sketch the graph of $y = f(x)$. Be sure to clearly label the vertex and any intercepts. You do NOT need to use transformations for this sketch.
 - State the maximum or minimum value of $f(x)$.

7. Let $h(x) = \frac{3-7x}{x-6}$. Find $h^{-1}(3)$.

8. For the angles a and b in the figures below, find the exact answer for $\cos(b - a)$, without using a calculator.



9. Verify the identity $\frac{2}{\tan(x) + \cot(x)} = \sin(2x)$.

10. Let $\sin(D) = \frac{5}{13}$, where D is in Quadrant II and $\cot(E) = \frac{3}{4}$, where E is in Quadrant III . Find:
- $\sec(D)$
 - $\cos(2E)$
 - $\sin(2D)$