QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 131 Fall 2022

Instructions: **Answer all questions.** **Show all work.**

1. Using algebraic methods, find each of the following limits. If a limit is ∞, $-\infty$, or does not exist, state this as your answer.
   a) $\lim_{x\to 2^-} \frac{2-x}{x^2-4}$
   b) $\lim_{x\to -1} \frac{|x|-1}{1+x}$
   c) $\lim_{x\to \infty} \frac{2x^3+4x^2+5}{1+7x^3}$
   d) $\lim_{u\to 0} \frac{u^2}{u+1}$
   e) $\lim_{x\to -1^+} \frac{3x^2-5}{1+x}$

2. a) State the definition that a function $f(x)$ is continuous at $x = a$.
   b) Let
   $$f(x) = \begin{cases} bx^2 + 1, & x \le 2, \\ \frac{1}{x+1}, & x > 2. \end{cases}$$
   Find the value(s) of b for which $f(x)$ is continuous on $\mathbb{R}$.

3. a) State the definition of $f'(-1)$, the derivative of a function $f(x)$, at $x = -1$.
   b) Using only this definition, compute $f'(-1)$ for $f(x) = \frac{1}{x}$.

4. Find $\frac{dy}{dx}$ for each of the following.
   a) $y = \frac{2x+3}{x^{1/2}+1}$
   b) $y = 2e^{-x} + \frac{4}{\sqrt{x}} + x^2 \cdot \ln(2x)$
   c) $xy^2 + \ln 3x = e^y + 2$

5. The weekly demand for a certain product is
   $$p(x) = 400 - 0.03x, \quad 0 \le x \le 10,000$$
   where $p(x)$ denotes the wholesale price in dollars and x denotes the quantity demanded. The weekly total cost function associated with manufacturing the product is given by
   $$C(x) = 0.0001x^3 - 0.02x^2 + 100x + 5000,$$
   where $C(x)$ denotes the total cost incurred in producing x units.

   a) Find the revenue and the profit functions.
   b) Find the marginal cost, the marginal revenue, and the marginal profit functions.
   c) Use marginal analysis to estimate the actual profit/loss realized from the sale of the 1001st unit.
   d) How many units, to the nearest integer, must be sold to maximize revenue?

(Continued on the back)

6. Let $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x + 3$.
   a) Find the intervals of increase or decrease of f.
   b) Find the local maximum and minimum values of f.
   c) Find the inflection points of f and the intervals where f is concave up or concave down.

7. How much money should be deposited in a bank paying interest at the rate of 2% compounded daily (assume a 365-day year) so that at the end of 10 years the accumulated amount will be $\$15,000$?

8. A certain radioactive element decays according to the law
$$Q(t) = Q_0 e^{-t/100},$$
where Q_0 is the initial size of the sample and t is the time measured in days. If the size of the sample left after 250 days is 15 mg, what was the initial size of the sample?

9. A cylindrical tank with radius of 3 feet and height of 12 feet is being filled with water at the rate of 2 cubic feet per minute. How fast is the water level rising when its height is 10 feet? (Hint: the volume V of a cylinder with height h and radius r is $V = \pi r^2 h$.)