Queens College
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Math 141 **Fall 2022**

Instructions: *Read each question on this exam before you start working so you can get the flavor of the questions. Please show all of your work. Unsupported answers will not even be graded. Do not cheat, else you pay with your academic life.*

1. Evaluate the following limits. If a limit is $\infty, -\infty$, or does not exist, show why without using a calculator.

   (a) $\lim_{x \to 4} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$.

   (b) $\lim_{h \to 0} \dfrac{(2+h)^3 - 8}{h}$.

   (c) $\lim_{x \to \infty} \dfrac{3x^4 - 2x^3 + 5}{-5x^4 + 3x^2 - 100}$.

   (d) $\lim_{x \to 0^-} \dfrac{|x|}{-x}$.

2. Using the definition of derivative (this involves a limit!), find $f'(x)$ where $f(x) = \frac{7}{x^2}$. Then, find an equation of the tangent line to the curve $f(x)$ at the point $(1, 7)$.

3. (a) Find the domain and range of the function $f(x) = \sqrt{16 - x^2}$. Write your answer using interval notation.

   (b) Find functions $f(x), g(x), h(x)$ such that $(f \circ g \circ h)(x) = \tan^4(\sqrt{x})$.

4. In each of the following, find $\frac{dy}{dx}$. (You need not simplify.)

   (a) $y = (2x^2 - 7x + 9)^5 \sin(x^2)$.

   (b) $y = \frac{\tan(x)}{1+\cos(x)}$.

   (c) $y = (x \tan(x))^{1/5}$.

   (d) $x \tan(y) = y - 1$.

5. (a) Show that the equation $4x + \sin(x) + 100 = 0$ has exactly one root between -30 and -20.

   (b) Use your graphing calculator to find the root in part (a), correct to three decimal places.

6. A car has position function
$$s(t) = t^3 \sin(t).$$

   (a) Determine the average velocity of the particle on the interval $[\pi/4, \pi/2]$.

   (b) Determine the instantaneous velocity of the particle at time $t = \pi/2$.

(continued on the back)

7. The volume of a cube is increasing at a rate of 5 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm?

8. Let $f(x) = x^4 - 4x^3 + 3$.

   (a) Find the intervals of increase and intervals of decrease of f.

   (b) Find the local maximum and minimum values of f, if any.

   (c) Find the intervals where f is concave up and those where f is concave down.

   (d) Find any and all inflection points of f.

   (e) Use the information found in parts (a) through (d) to sketch the graph of $y = f(x)$.

9. (a) State the Mean Value Theorem.

   (b) Using the function given by $f(x) = x^3 - 3x + 2$, find all numbers c that satisfy the conclusion of the Mean Value Theorem on the interval $[-2, 2]$.

10. Find two positive integers such that the sum of the first number and four times the second number is 1000, and the product of the numbers is as large as possible.