QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Math 142 **Fall 2022**

Instructions. Answer each question and show your work.

1. Find the derivative $\frac{dy}{dx}$ for each of the following functions. Simplify if possible.

   a) $y = \tan^{-1} 3x$

   b) $y = x^{2x}$

   c) $y = \ln \dfrac{(2x+1)(3x+1)}{\sqrt{4x+1}}$

   d) $y = \displaystyle\int_{-1}^{\cos x} \sqrt{t^2+1}\ dt$

2. Compute each of the following integrals without the use of a calculator.

   a) $\displaystyle\int \frac{3}{\sqrt{1-4x^2}}dx$

   b) $\displaystyle\int \frac{e^{2x}+e^{3x}}{e^{5x}}dx$

   c) $\displaystyle\int_1^2 x\sin(\pi x^2)\,dx$

   d) $\displaystyle\int_1^2 (3x+2\sqrt{x})(3x-2\sqrt{x})\,dx$

   e) $\displaystyle\int_1^e \frac{1}{x[\ln(x)+1]}\,dx$ (Express your answer in terms of ln or e if needed.)

3. When an object is thrown straight up from a height of 512 feet with an initial velocity of 64 feet per second, the only force acting on the body is gravity in the downward direction. Using the ground level as the origin, use calculus to
   a) find the velocity of the object at any time.
   b) find the height of the object above ground at any time t.
   c) determine the time when the object hits the ground.
   d) determine the speed at which it hits the ground.

4. A bacteria culture initially contains 500 cells and grows at a rate proportional to its size. After 2 hours, its population (i.e., the number of cells) has increased to 12,500.
   a) Find an expression for the bacteria population after t hours.
   b) When will the population reach 1,000,000 (one million)? (Express your answer in terms of ln or log if needed.)

(Continued on the back)

5. Consider the curve $y^2 = 4(x+1)^3$ where $0 \le x \le 3,\ y \ge 0$. Find the length of the curve. Express your answer in terms of square roots if needed.

6. Let R be the region in the xy-plane bounded by the curves $y = x^3 - 4x$ and $y = -3x$ where $x \ge 0$.
   a) Sketch the curves and label their intersection(s) for $x \ge 0$.
   b) Find the area of R.
   c) Find the volume of the solid of revolution obtained by rotating R about the x-axis. Express your answer in terms of π as needed.
   d) Find the volume of the solid of revolution obtained by rotating R about the y-axis. Express your answer in terms of π as needed.

7. Find the solution of the initial value problem $\frac{dy}{dx} = \frac{4xy}{x^2+1}$ with the initial condition $y(0) = 5$.

8. In this problem, the three parts (a), (b), and (c) are independent. Therefore, your answer in one part will not impact the other parts.
   a) Express the following as an integral:

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} (x_i^2 - 4x_i)\Delta x$$

   where the x_i's are the right endpoints of a partition of the interval $[2,4]$.

   b) Let $f(x)$ be a function such that $\int_2^7 f'(x)dx = 6$ and $f(7) = 5$. Find $f(2)$.

   c) Below is the graph of a function $h(x)$,which consists of a line segment over [0,3], and two semicircles over [3,7] and [7,9] respectively. Compute

$$\int_0^9 (h(x) + 2x)\, dx\ .$$