

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
 $2\frac{1}{2}$ HOURS

Mathematics 143

Fall 2022

Instructions: Answer all questions. Show all work.

1) Evaluate:

a) $\int x^2 \sin x \, dx$

b) $\int \cos^4 x \, dx$

c) $\int \sec^4 x \tan^2 x \, dx$

d) $\int x^5(1 - x^2)^{1/2} \, dx$

e) $\int_0^2 \frac{x^3}{(4 + x^2)^{1/2}} \, dx$

f) $\int \frac{3x^2 - 4x + 6}{(x - 2)(x^2 + 1)} \, dx$

g) $\lim_{x \rightarrow \infty} \left[1 + \frac{1}{x^2}\right]^x$

2) Determine whether the improper integral $\int_0^{\infty} x e^{-3x} \, dx$ converges or diverges. If it is convergent, find its value.

3) Determine whether the following series converge or diverge:

a) $\sum_{n=1}^{\infty} \frac{2n}{n^3 - 4}$

b) $\sum_{n=1}^{\infty} \frac{1 + \sin(2n)}{n^2 + 1}$

4) Find the radius and the interval of convergence for each of the following:

a) $\sum_{n=1}^{\infty} \frac{(x + 1)^n}{n \cdot 3^n}$

b) $\sum_{n=1}^{\infty} \frac{n^2 x^n}{1 + 2n}$

5) Using the definition alone, find the Maclaurin series for $f(x) = \ln(1 - 2x)$ and determine its interval of convergence.

6) Compute $\int_0^1 \frac{\cos x - 1}{x^2} \, dx$ with an error of at most 0.001.

7) Let $f(x) = x^{1/3}$, and $a = 8$.

a) Compute $T_2(x)$, the second Taylor polynomial of f about 8.

b) What is the largest that $|f(x) - T_2(x)|$ can be if $7 \leq x \leq 9$?