QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 151 **Fall 2022**

Instructions: Answer all questions. Show all work.

1) Evaluate each of the following limits. If limit is $+\infty$, $-\infty$ or does not exist, state this as your answer.

a) $\lim_{x \to -7} \dfrac{\frac{1}{7}+\frac{1}{x}}{x+7}$

b) $\lim_{x \to 0} \dfrac{\tan(2x)}{4x}$

2) a) Use the definition of the derivative to find $f'(x)$ if $f(x) = \sqrt{4-x}$.
b) Using the result of part a), write an equation of the tangent line to the graph of $y = f(x)$ at the point $P(3,1)$.

3) a) An object moves along a horizontal line according to the position function

$$s = f(t) = 4\sqrt{t} - \frac{6}{\sqrt[3]{t}} + 18\sqrt[6]{t},$$

where t (> 0) is in seconds and s is in feet. Find the velocity and the acceleration of the object one second after it begins to move.

b) If $g(x) = \sin x \tan x - \cos^2(2x)$. Find $g'\left(\frac{\pi}{6}\right)$.

4) Let $f(x) = \dfrac{-4x}{x^2+4}$.

a) Find the intervals on which f is increasing and those on which is f decreasing.
b) Find the x-coordinates of any and all relative extrema of f. Identify each as either a relative maximum or a relative minimum of f.
c) Find the intervals on which f is concave upward and those on which is f concave downward.
d) Find the x-coordinates of any and all points of inflection of f.
e) Find any and all horizontal and vertical asymptotes of f. Justify each by an appropriate limit.
f) Using the results of parts a) – e), sketch the graph of f.

5) Let $P(x,y)$ be a point of the graph of $y^2 = 4 - 2x$ with the property that the square of the distance between P and $Q(-2,0)$ is as small as possible. Find all such points P. (Hint: The square of the distance between the points (x_1, y_1) and (x_2, y_2) is $(x_2 - x_1)^2 + (y_2 - y_1)^2$.)

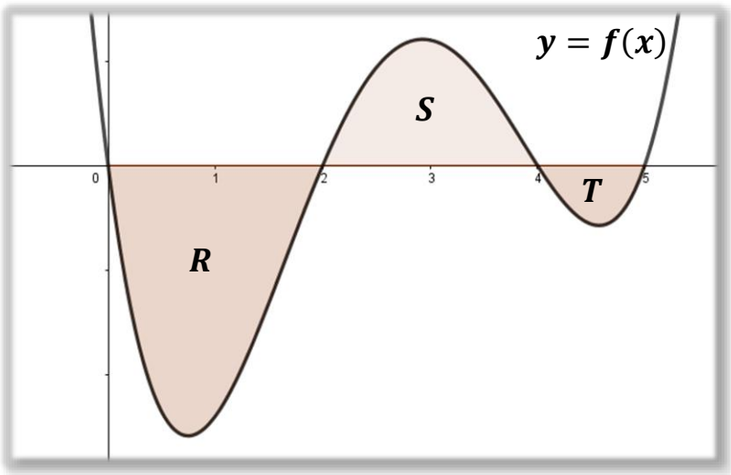
6) Let R be the region in the xy-plane bounded above by the graph of $y = f(x) = \cos x$ and below by the x-axis, where $0 \le x \le \frac{\pi}{3}$.

a) Using right endpoints, write an expression that can be used to approximate A, the area of R, using four rectangles.
b) Using an appropriate definite integral to compute the exact value of A.

(Continued on the back)

7) An airplane flying horizontally with respect to the ground at an altitude of 1 mile passes directly over a radar station. If the speed of the airplane is 500 miles per hour, find the rate at which the distance between the airplane and the radar station is increasing when the airplane is 2 miles away from the radar station.

8) The graph of a continuous function f is shown below:



The area of the regions R, S, and T are as follows: Area of $R = 16$, Area of $S = 8$, and Area of $T = 2$. Use the properties of definite integrals to evaluate each of the following:

a) $\int_0^5 f(x)\ dx$

b) the average value of f on $[0,5]$

c) $\int_2^0 (f(x) + 3)\ dx$

d) $\int_2^4 \left(4f(x) + \frac{1}{x^2}\right) dx$

e) $g'(4)$, where $g(x) = \int_0^x f(t)\ dt$

9) a) Find the most general antiderivative of $\frac{x - x^2}{2\sqrt[3]{x}}$, where $x \neq 0$.

b) Find the exact value of each of the following definite integrals:

(i) $\int_{\pi/4}^{\pi/2} (2 - \csc^2 x)\ dx$

(ii) $\int_0^2 \frac{x}{\sqrt{1 + 2x^2}}\ dx$