Instructions. Answer all questions.

Part I: Matching

The answers to problems $1-12$ (out of order) are:

$$0, \quad \frac{1}{e}, \quad \ln(2), \quad \frac{1}{\sqrt{2}}, \quad \frac{\pi}{4}, \quad 1, \quad 1, \quad \frac{\pi}{2}, \quad e-1, \quad 2, \quad 2, \quad \infty$$

1. $\displaystyle\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n =$

2. $\displaystyle\lim_{n\to\infty}\left(\frac{1}{n}\right)^n =$

3. $\displaystyle\int_0^\infty \frac{x^2}{e^x}\,dx =$

4. $\displaystyle\sum_{k=1}^\infty \frac{1}{k!} =$

5. $\displaystyle\sum_{k=1}^\infty \frac{1}{k} =$

6. $\displaystyle\sum_{k=1}^\infty \frac{1}{k+k^2} =$

7. $\displaystyle\sum_{n=0}^\infty (-1)^n\left(\frac{\pi}{4}\right)^{2n+1}\frac{1}{(2n+1)!} =$

8. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx =$

9. $\displaystyle\int_1^\infty \frac{1}{1+x^2}\,dx =$

10. $\displaystyle\int_1^\infty \frac{1}{x+x^2}\,dx =$

11. $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}dx =$

12. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}dx =$

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Part II: Multiple Choice

13. Let $\{s_n\}$ be the sequence of partial sums for the series $\sum_{k=1}^{\infty} \frac{k^2(k+3)!}{(2k)!}$. Then $s_4 =$

(a) $\frac{31}{105}$

(b) $\frac{569}{2520}$

(c) $\frac{20}{7}$

(d) $\frac{429}{12}$

(e) 43

14. $\cos(\arctan(2)) =$

(a) $\frac{1}{2}$

(b) $\frac{1}{\sqrt{5}}$

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{2}{\sqrt{3}}$

(e) undefined

15. If $\sinh(x) = \frac{e^x - e^{-x}}{2}$ and $\cosh(x) = \frac{e^x + e^{-x}}{2}$ then which *one* the following statements is false:

(a) $\sinh(0) = 0$

(b) $\cosh(0) = 1$

(c) $\frac{d}{dx}\sinh(x) = \cosh(x)$

(d) $\frac{d}{dx}\cosh(x) = -\sinh(x)$

(e) $\lim_{x\to 0} \frac{\sinh(x)}{x} = 1$

(f) $\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$

16. The interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{3^k\sqrt{k}}$ is

(a) $-2 < x < 4$

(b) $-4 < x \le 2$

(c) $2 < x \le 4$

(d) $-2 \le x < 4$

(e) $-4 \le x < 2$

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Part III: Infinite Series

Determine whether the following series converge absolutely, converge conditionally, or diverge. Justify your answers.

17. $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{2k+1}{\sqrt{k^2+4}}\right)$

18. $\displaystyle\sum_{k=1}^{\infty} \frac{\cos(k^{10})}{10^k}$

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Part IV: Applications of Power Series

The Maclaurin series for $g(x) = x^2 e^{-x^2}$ is given by

$$g(x) = x^2 - x^4 + \frac{x^6}{2} - \frac{x^8}{6} + \frac{x^{10}}{24} - \frac{x^{12}}{120} + \cdots$$

Use it to solve the next three problems.

19. $g^{(6)}(0) =$

(a) -20

(b) $-\frac{1}{720}$

(c) $-\frac{1}{6}$

(d) $\frac{1}{2}$

(e) 360

20. $\lim_{x\to 0} \frac{x^2 e^{-x^2} - x^2 + x^4}{x^6} =$

(a) ∞

(b) 0

(c) -2

(d) $-\infty$

(e) $\frac{1}{2}$

21. An approximation for $\int_0^{\frac{1}{2}} x^2 e^{-x^2}\,dx$ with an error less than $\frac{1}{1792}$ is

(a) $\frac{17}{480}$

(b) $\frac{25}{128}$

(c) 0.00040318

(d) $\frac{193}{21420}$

(e) $\frac{193}{1260}$

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