MATH 120 — Final Exam — 19 May 2023

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Important Information:

WAIT until you are told to start.

For the counting questions, leave your answers in terms of $n!$, $P(n,k)$, $\binom{n}{k}$, and $\left(\!\binom{n}{k}\!\right)$.

You are expected to SHOW YOUR WORK in all answers.
Wrong answers provided without work will receive no credit.

You may use the back of pages if you need more space or for scrap work.

If you continue work in a different location and want it to be graded, indicate CLEARLY where it is.

1. When we write the expression "$f : A \to B$" in symbols,

   (a) What is f?

   (b) What is A?

   (c) What is B?

   (d) How would you say this expression in words?

2. Consider the set $T = \left\{ \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots \right\}$

   (a) Write T in set-builder notation.

   (b) Is $\frac{1}{144} \in T$? Why or why not?

3. Consider the universe $\mathcal{U}$ to be the set of Queens College students. Some Queens College students are taking MATH 120. Some Queens College students are taking MATH 141.

   (a) Draw and **shade in** a Venn Diagram that shows the set of students S who are taking **at most one** of MATH 120 and MATH 141.

   (b) How would you describe **in words** the set of students in the **complement** of S?

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4. The National Football League has 32 teams, including the New York Jets and New York Giants.

   (a) In how many ways can the League choose two teams to play in the first game of the season?

   (b) In how many ways can the New York Jets play ten different opponents in their first ten games of the season? (Here, schedule order matters, so playing Team A in game 1 and Team B in game 2 is different from playing Team B in game 1 then Team A in game 2.)

   (c) This year, the New York Giants will play the following teams.

   - They will play the following three teams exactly two times each.
     Philadelphia $\times 2$, Dallas $\times 2$, Washington $\times 2$.
   - They will play the following eleven teams exactly once each.
     Green Bay $\times 1$, NY Jets $\times 1$, New England $\times 1$, LA Rams $\times 1$, Seattle $\times 1$, Arizona $\times 1$, New Orleans $\times 1$, Las Vegas $\times 1$, San Francisco $\times 1$, Miami $\times 1$, Buffalo $\times 1$.

   How many different schedules (of 17 games) are possible for the New York Giants' season?

   (Again, order matters. Ignore the location where the games are played (home vs. away) and ignore that there is a "bye" week when teams do not play.)

5. Consider the set P of permutations of $\{1, 2, \ldots, 8\}$.
   Let $Q \subseteq P$ be those permutations that do not start with 8.
   Let $R \subseteq P$ be those permutations where 1, 2, 3, and 4 come before 5, 6, 7, and 8.

   (a) Determine $|Q|$.

   (b) Determine $|R|$.

   (c) Is $R \subseteq Q$? Draw a Venn Diagram that shows the relationship between P, Q, and R.

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6. Let N be the set of even integers and let M be the integer multiples of 3.
Define a function f that takes as inputs $n \in N$ and $m \in M$ and outputs $n + m$.

   (a) What is the domain of f?

   (b) What is a codomain of f?

   (c) Is 11 in the range of f?

   (d) Is f injective? Why or why not?

7. Suppose $g : X \to Y$ is a function and $Z \subseteq Y$. Describe in words what type of object the quantity $g^{-1}(Z)$ is and how you could carefully describe that object.

8. Consider the sequence s defined by $s_n = \left\lfloor \frac{n}{2} \right\rfloor$. Compute its first eight terms $(s_0, s_1, s_2, \ldots, s_7)$.

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9. Write $\dfrac{e^5 \cdot e^6 \cdot (e^2)^4}{e^{-1}\sqrt{e}}$ as a power of e.

10. Write $\log_2(7)$ as an expression involving base 10 logarithms only.

11. Compute the sum $-8 - 3 + 2 + 7 + \cdots + 1002$. Show your work.

12. Write out $\displaystyle\prod_{i=567}^{1234} \frac{1}{i-2}$ as a product of factors.

13. Convert $\displaystyle\prod_{i=0}^{99}(ar^i)$ to an expression involving summation notation.

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14. If p, q, and r are distinct prime numbers and $a = p^{12} \cdot q^{11} \cdot r^{10}$,

    (a) How many factors does a have?

    (b) Is $b = p^6 \cdot q^8 \cdot r^{10}$ a factor of a? Why or why not?

15. Consider the numbers $c = 6240$ and $d = 750$.

    (a) Determine the prime factorizations of c and d.

    (b) Use this information to determine the LCM of these two numbers. Leave your answers in terms of its prime factorization.

16. Use the Euclidean Algorithm to compute $\gcd(336, 434)$.

17. Use the technique of repeated squaring to find the last digit of 87^{26}.

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