QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 110   **Spring 2023**

<u>Instructions</u>: Answer all questions. Show all work.

1.   If 627 votes are cast, what is the smallest number of votes a winning candidate can have in a 7 candidate race to be decided by plurality?

2.   150 votes are cast in an election for best mathematician, to be decided by plurality. The tallies so far are:

Euclid	30
Archimedes	26
Leonhard Euler	19
Isaac Newton	15
Carl Friedrich Gauss	10

   a)   What is the minimal number of remaining votes Euclid needs to be assured of a win?
   b)   What is the minimal number of remaining votes Newton needs to be assured of a win?

3.   The preference rankings of 22 students are:

	<u>6</u>	<u>6</u>	<u>3</u>	<u>1</u>	<u>2</u>	<u>2</u>	<u>1</u>	<u>1</u>
Basketball	1√	2√	1√	2	2√	3	3√	3
Soccer	2√	3√	3√	1√	1√	1√	2√	4
Tennis	4	4√	4	3	4	4	1√	1√
Volleyball	3	1√	2√	4	3√	2√	4	2

   a)   Which sport wins by plurality?
   b)   Which sport wins by plurality with runoff between the top two finishers?
   c)   Which sport has the top Borda count?
   d)   Which sport is the Condorcet winner?
   e)   Which sport wins the approval vote?

4.   Alex, Bob, and Carlos invest in an ocean dive seeking pirate treasure.

<u>Investor</u>	<u>Dollars Invested</u>
Alex	7,600
Bob	5,900
Carlos	1,400

   The dive results in 40 gold coins. Apportion the coins to the investors using:
   a)   Hamilton's method.
   b)   Lowndes' method.
   c)   Jefferson's method.

5.   The grades on an algebra exam are as follows for a class:
        87   72   72   83   85   81   97   62   90

        a)   Using the ungrouped data, find the mean, median, mode, range, and standard deviation.
        b)   Construct a box-and-whisker plot.
        c)   Construct a histogram with first interval 60-69.
        d)   What is the modal class?

6.   a)   How many unique 5-digit zip codes can be made from the 10 digits 0 – 9 if no digit can be repeated?

        b)   How many unique 5-digit zip codes can be made from the 10 digits 0 – 9 if the digits can be repeated and the first digit cannot be zero?

7.   An experiment has outcomes 0, 1, 2, 3, and 4 with probabilities as shown:

$p(x)$	.42	.05	.2	?	.08
x	0	1	2	3	4

        a)   Find the missing probability.
        b)   Calculate the mean.
        c)   Construct the probability histogram.
        d)   What is the probability of an outcome less than 3?

8.   Three cards are selected from a standard 52-card deck. (A standard 52-card deck has 4 suits – clubs, diamonds, hearts and spades – each with 13 cards: 2, 3, 4, … 10, J, Q, K, A.) What is the probability all three are kings if
        a)   there is replacement?
        b)   there is no replacement?

9.   The salaries of employees at a company are normally distributed with a mean of $ 50,000 and standard deviation of $ 20,000.
        a)   What percentage earn less than $ 50,000?
        b)   What percentage earn less than $ 40,000?
        c)   Below what amount are 88.69% of salaries?
        d)   If there are 5,000 employees, how many earn more than $ 100,000?

Statistical Tables

Table A Normal Curve (z) Table

The normal curve table gives only the percentage of data starting from the middle ($z = 0$), out to whatever z score you look up. For instance, if you look up $z = 1.28$, you get .3997. This means .3997 or 39.97% of the data in the normal curve is found between $z = 0$ and $z = 1.28$.
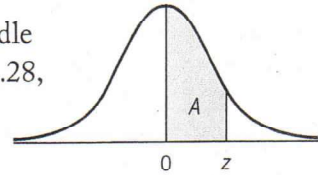
					Normal					
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998