MATH 115 DEPARTMENT FINAL EXAM – SPRING 2023
Time Limit: 2 hours and 30 minutes

Directions: Answer all questions in a **neat and coherent way** and show all work in the provided blue book. If no answer is given, the question will be marked wrong. **Mysterious or unsupported answers will not receive credit. Any additional help is prohibited. If you are found cheating, you will be granted a zero and asked to leave.**

1. Given the points $P(4, -4)$, $Q(2,6)$, and $T(7,8)$:
   a. Find an equation of a line containing the points P and Q.
   b. Find an equation of a line parallel to $\overline{PQ}$ that passes through point T.
   c. Find an equation of a line perpendicular to $\overline{QT}$ that passes through P.
   d. Find the midpoint of $\overline{QT}$.
   e. What is the length of the line segment of $\overline{PT}$?

2. Given $f(x) = 3x - 4$, $g(x) = 2x - 5$, find the following in **simplest** form:
   a. $f(2 - h) - f(2)$
   b. $2g(x) + [f(x)]^2$
   c. $4f(0) - g(1)$

3. Find the domain of each of the following functions and state the domain using either **set building** or **interval notation**.
   a. $g(x) = \dfrac{2x - 1}{x^2 + x - 6}$
   b. $f(x) = 3x^2 + 8x - 4$
   c. $h(x) = \dfrac{1}{\sqrt{x - 7}}$

4. The AMC Fresh Meadows movie theater charges \$5 for children's tickets and \$9 for adult tickets. How many tickets of each type were sold to the new Marvel Guardians of the Galaxy movie if 70 tickets were sold and the receipts totaled to \$550?

5. Simplify:
   a. $\dfrac{-3ab^2}{(9a^2b^4)^3}$
   b. $\dfrac{(2^{-1}x^1y^{-3})^{-2}}{4^{-1}x^{-3}y^{-5}}$
   c. $2\sqrt{32x^2y^3} - xy\sqrt{98y}$

6. Simplify the following expressions. All answers should be expressed in **simplest** form.
   a. $\dfrac{6x^2y^4}{35a^2b^5} \div \dfrac{12x^3y^3}{7a^4b^5}$
   b. $\dfrac{2x^2 + 4x}{8x^2 - 40x} \cdot \dfrac{6x^3 - 30x^2}{3x^2 + 6x}$
   c. $\dfrac{x^2 + 4}{4x^2 - 36} - \dfrac{4}{x + 3}$
   d. $\dfrac{\dfrac{3}{x} + \dfrac{11}{x^2} - \dfrac{4}{x^3}}{1 + \dfrac{2}{x} - \dfrac{8}{x^2}}$

(continued on the back)

7.   Rationalize the denominators and simplify.
$$\frac{5}{\sqrt{3}-2} - \frac{6}{\sqrt{3}}$$

8.   Final all real solutions for each of the following equations:
a. $\sqrt[3]{x-4} + 7 = 5$
b. $-x^2 + 3x + 5 = 0$
c. $\sqrt{10 - 3x} + x = 2$
d. $4(x-2)^2 - 36 = 0$
e. $2x^2 + 14x = -24$

9.   Solve the following system of linear equations and state whether the system is independent, inconsistent, or dependent.
$$\begin{cases} 2x - 5y = -16 \\ 3x + 2y = -5 \end{cases}$$

10.   Sketch and label the graph of $2x + 3y = -3$ using its x and y-intercepts. Be sure to label the axes, the graph, and the x- and y-intercepts.

11.   Given the parabola whose equation is $y = f(x) = 3x^2 - 2x$, find each of the following:
a.   The coordinates of its x-intercept(s).
b.   The coordinates of its y-intercept.
c.   The coordinates of its vertex.
d.   An equation of its axis of symmetry.

12.   Find the quotient when $-10 - 33x + 3x^3 - 8x^2$ is divided by $3x + 1$.

13.   Factor each of the following polynomials completely:
a. $t^2 + 3t - 18$
b. $6x^5 + 74x^4 + 24x^3$
c. $x^4 - 16$

14.   Find the zeros of the polynomial $12x + x^2 - 6x^3$.

Suggested point values for each question:
- Questions 4, 6, and 8: 4 points each part.
- Questions 1, 5, 10, 12, and 14: 3 points each part.
- Questions 2, 7, 9, 11, and 13: 2 points each part.
- Question 3: 1 point each part.