MATH 120 — Final Exam — 12 Dec 2023

Name: ________________________________

Class Section: ____________________________

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Important Information:

WAIT until you are told to start.

For the counting questions, leave your answers in terms of $n!$, $P(n,k)$, $\binom{n}{k}$, and $\left(\!\binom{n}{k}\!\right)$.

You are expected to SHOW YOUR WORK in all answers.
Wrong answers provided without work will receive no credit.

You may use the back of pages if you need more space or for scrap work.

If you continue work in a different location and want it to be graded,
indicate CLEARLY where it is.

1. (20 pts) Complete the definition exercise for a **surjective** function:

   (a) Recall the precise definition a *surjective* function is:

   A function $f : X \to Y$ is *surjective* if for every element $y \in Y$, there exists at least one element $x \in X$ such that $f(x) = y$.

   (b) Give your informal understanding of when a function is *surjective.*

   (c) Give an example of a function that **IS** *surjective.* (And explain why.)

   (d) Give an example of a function that **IS NOT** *surjective.* (And explain why.)

2. (30 pts) Consider the set M of natural numbers that are multiples of 2 but not multiples of 3.

   (a) Write M in roster notation.

   (b) Is $60 \in M$? Why or why not?

   (c) Is $M \subseteq \mathbb{Z}$? Why or why not?

3. (10 pts) Write the set $P = \{-88, -77, -66, \ldots, 44, 55\}$ in set-builder notation.

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4. (10 pts) Draw a Venn diagram with three circles to represent the set of people who may or may not have unicycles, bicycles, and/or scooters. Shade in all the region(s) of the diagram that correspond to "people who have unicycles, bicycles, or scooters, but not all three"'.

   [For clarity, the OR is an inclusive OR]

5. (20 pts) For each of the following expressions, determine whether it is True, False, or Invalid.

   You are encouraged to add a sentence of explanation or a counterexample. This may allow you to receive partial credit if you convey an understanding of the situation.

   (a) $88 \mid 22$

   (b) $100 \equiv 1000$

   (c) For all $x \in \mathbb{R}$, $\lfloor x + \frac{1}{2} \rfloor = \lceil x \rceil$

   (d) $0! = 0$

6. (20 pts) Consider the sets $A = \{a, e, i, o, u\}$, $B = \{b, c, d\}$, $C = \{f, g, h\}$, and $D = \{w, x, y, z\}$.

   (a) In how many ways can you choose one element from any of the sets A, B, C, or D?

   (b) In how many ways can you create a four-letter password that has exactly one letter from A, one letter from B, one letter from C, and one letter from D, **in any order**? Note that order matters, so '`qrst`' is different from '`rqts`'.

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7. (30 pts) You enter “Insomnia Cookies” in search of a late night snack. They have 8 kinds of cookies on the menu, including one kind called “Chocolate Chunk”.

   (a) You decide to buy eight cookies — one cookie of each kind.
   In how many different orders could you eat all eight cookies?

   (b) Suppose you may order multiple cookies of the same type. In how many ways can you buy a box of six cookies if exactly two of them are “Chocolate Chunk”?

   (c) In how many ways can you buy a box of six cookies that are not all the same kind?

8. (20 pts) Consider the set B of bit strings of length 10 that have exactly 7 zeros.

   (a) Give an example of an element of B.

   (b) How many elements does B have?

9. (20 pts) Define a function n that takes as input a person born in the United States and gives as output the first letter of their last name on their birth certificate.

   (a) Determine the domain and codomain of this function.

   (b) Is this function injective? Why or why not?

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10. (20 pts) Define a rule r that associates to every classroom at Queens College the number of people currently in that room.

   (a) Show that r is a **well-defined function** from the set of classrooms to $\mathbb{N}$.

   (b) Describe in words what $r^{-1}(0)$ is.

11. (20 pts)

   (a) Use rules of exponents to write $\dfrac{9^{10}\sqrt{3}}{3^{-3}}$ as $(\text{something})^{(\text{something})}$.

   (b) Use rules of logarithms to write $\log_2(w+x)+3\log_2(y)-4\log_2(z)$ as $\log_2(\text{something})$

12. (20 pts) Consider the sum $S=\left(1\frac{1}{2}\right)+\left(2\frac{1}{4}\right)+\left(3\frac{1}{8}\right)+\cdots+\left(10\frac{1}{1024}\right)$.

   (a) Write S in sigma notation.

   (b) Use techniques from class to compute the value of S.

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13. (10 pts) Consider the logarithmic expression $\log_2\left(\prod_{i=1}^{30}(i+i^2)\right)$.

Rewrite this as an expression involving sigma notation.

14. (10 pts) Calculate $8+8^2+8^4+8^8+8^{16}+8^{32}+8^{64}$ modulo 11.

15. (10 pts) Use the Euclidean Algorithm to find $\gcd(3094, 819)$.

16. (30 pts) The numbers $a = 324$ and $b = 384$ are both numbers of the form $2^m \cdot 3^n$ for some $m, n \in \mathbb{Z}$.

(a) Find the prime factorizations of a and b.

(b) How many divisors does a have? Explain your answer.

(c) Use the prime factorizations of a and b to find $\operatorname{lcm}(a, b)$. Explain your answer.

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