

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

**Final Examination
2 $\frac{1}{2}$ Hours**

Mathematics 122

Spring 2023

Instructions: Answer all questions. Show all work.

1. Find the domain of each of the following functions. Express your answer in interval notation.

a) $f(x) = \frac{x - 5}{\sqrt{3x - 6}}$

b) $g(x) = \frac{\sqrt{3x - 6}}{x - 5}$

c) $h(x) = \frac{x - 5}{3x^2 + 5x - 2}$

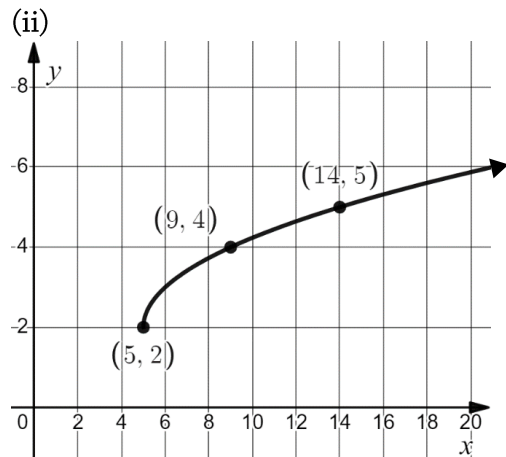
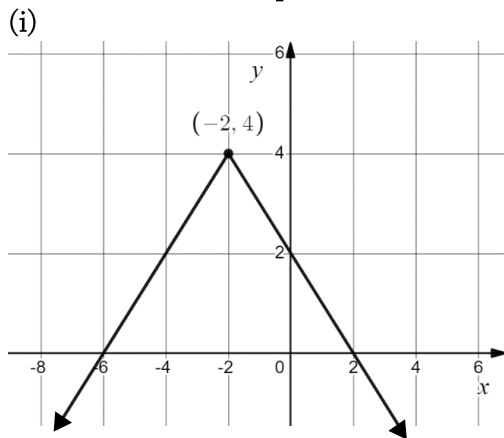
2. Let $f(x) = 3x^2 - 5x + 7$. Find and simplify:

a) $f(a)$

b) $f(a + h)$

c) $\frac{f(a + h) - f(a)}{h}$ ($h \neq 0$)

3. a) Write an equation for each of the following graphs (each of which represents a transformation of a parent function):



b) Sketch the graph of each of the following functions. Label the coordinates of any x - and y -intercepts. Write the equations of the vertical and horizontal asymptotes, where appropriate.

(i) $y = 1 - (x - 2)^3$ (Describe the “end behavior”.)

(ii) $y = 3 \sin(2x)$ in the interval $[0, 2\pi]$

(iii) $y = \frac{1}{x + 2} - 3$

4. Let $f(x) = 5 - 6x^2$ and $g(x) = 3x - 4$.

a) Evaluate $(g \circ f)(-3)$.

b) Find $(f \circ g)(x)$ and simplify.

(continued on the back)

5. Let $f(x) = \frac{5x - 2}{3x + 4}$.
- Find $f^{-1}(x)$ and its domain. (Express in interval notation.)
 - Compute $f^{-1}(-1)$.
6. Let $f(x) = 2x^2 + 12x + 10$
- Express $f(x)$ in standard vertex form.
 - Sketch the graph of $y = f(x)$ and clearly label the coordinates of the vertex and any x - and y -intercepts.
 - State the maximum or minimum value of $f(x)$.
7. Solve each of the following for x . Round each answer to four decimal places when necessary.
- $27^{3x-2} = 81^{x+1}$
 - $\log_{12}(x - 6) + \log_{12}(x + 1) = 2$
 - $6 + 2e^{3-x} = 7$
8. Without using your calculator, evaluate the following expressions. Show all work necessary:
- $\log_8\left(\frac{1}{4}\right)$
 - $\log_{20} 200 + \log_{20} 50 - \log_{20} 25$
 - $\sin\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{12}\right) + \cos\left(\frac{\pi}{6}\right) \sin\left(\frac{\pi}{12}\right)$
 - $\sec\left(\tan^{-1}\left(\frac{8}{15}\right)\right)$
9. John invests \$ 5,000 in an account paying 11% interest compounded continuously.
- How much money will be in his account after 8 years? (Round the answer to closest cent.)
 - How long will it take for his initial amount of \$ 5,000 to be doubled? (Round the answer to one decimal place.)
10. If $\sin A = \frac{3}{5}$ where $\angle A$ is in Quadrant II and $\cot B = -\frac{12}{5}$ where $\angle B$ is in Quadrant IV, find:
- $\sec A$
 - $\sin 2B$
 - $\cos(A - B)$
 - $\cos^2 A$
11. Verify the identity: $\tan x + \cot x = \cot x \sec^2 x$
12. Solve for x on the interval $[0, 2\pi)$: $-3 \cos x + 3 = 2 \sin^2 x$