

QUEENS COLLEGE  
DEPARTMENT OF MATHEMATICS

Final Examination  
2 ½ hours

Mathematics 131

Spring 2023

Instructions: Please answer all questions and show all your work in the blue book provided.

- 1) Use analytical methods (not your calculator) to find each of the following limits. If the limit is  $+\infty$ ,  $-\infty$  or does not exist, explain why

a. 
$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{e^x}$$

b. 
$$\lim_{x \rightarrow 4^+} \frac{16-x^2}{x^2-5x+4}$$

c. 
$$\lim_{x \rightarrow \infty} \frac{3x^3-5x^2+7x}{8-11x^3}$$

d. 
$$\lim_{x \rightarrow 5^-} \frac{x+3}{5-x}$$

- 2) Create a table (using your calculator) to approximate

$$\lim_{x \rightarrow 1} \frac{3^x - 3}{4^x - 4}$$

Please make sure to include smaller and larger values of  $x$  approaching 1 and a final approximation of the limit correct to 3 decimal places.

- 3) Let  $f(x) = 2x^2 - 4x + 1$ .

- a. Use the **definition of the derivative** to find  $f'(x)$ .  
b. Find an equation of the tangent line at the point  $(3, 7)$ .

- 4) Find  $\frac{dy}{dx}$  for each of the following. (You do not need to simplify.)

a. 
$$y = \frac{4}{3}x^3 - \frac{1}{7x^2} + \sqrt[5]{x^3} + \ln(3x^3 - 4) + e^7$$

b. 
$$y = \sqrt{x^3 - 4x} \cdot (x^4 + 3)^2$$

c. 
$$y = \frac{e^{3x}}{(4x-1)^3}$$

d. 
$$y = (x^3 + 4x)^{5x}$$
 (Hint: use logarithmic differentiation)

e. 
$$y^3 + 5x^2y = 6 - x + \sqrt{y}$$

- 5) Let  $f(x) = x^3 - 6x^2 + 4$ .

- a. Is  $f(x)$  a continuous function? Explain.  
b. Can the Intermediate Value Theorem be used to show that  $f(x)$  has a zero on the interval  $[0, 3]$ ? Explain.  
c. Can the Intermediate Value Theorem be used to show that there is some  $c$  on the interval  $[0, 3]$  such that  $f(c) = -7$ ? Explain.

(continued on the back)

- 6) Let  $f(x) = 2x^4 - 4x^2 - 16$
- Find the intervals of increase and decrease of  $f$ .
  - Find the relative (local) maxima and minima of  $f$  and their coordinates.
  - Find the intervals of upward and downward concavity of  $f$ .
  - Find the inflection points of  $f$ .
  - Graph  $f(x)$  using the information from parts a-d and label any important points.
- 7) The demand for  $x$  number of calculators is given by the price function  $p(x) = -\frac{1}{3}x^2 - 8x + 2227$  and the total cost function for  $x$  number of calculators is given by  $C(x) = x^2 + 4x + 39$ .
- Find the total revenue function  $R(x)$ .
  - Find the total profit function  $P(x)$ .
  - Use calculus to find how many calculators would generate the maximum profit? What is the maximum profit?
- 8) The total cost of a certain product is given by  $C(x) = .002x^3 - .39x^2 + 18x + 200$  for  $0 \leq x \leq 175$ .
- Use marginal analysis to estimate the cost of producing the 125<sup>th</sup> product.
  - Find the actual cost of producing the 125<sup>th</sup> product.
  - Find the critical number(s) of the cost function.
  - Find the absolute minimum cost and absolute maximum cost in the specified interval given above.
- 9) The demand function for compasses is given by  $39x^2 + 9p^2 = 3600$  where  $x$  represents the number of compasses (measured in hundreds) and  $p$  is the unit price measured in dollars. How fast is the quantity of compasses changing when the unit price is \$7 and increasing at a rate of \$0.38/month?
- 10) Suppose that \$3500 is invested at the interest rate of 1.7% per year.
- Compute the balance after 5 years if interest is compounded quarterly.
  - Compute the balance after 5 years if interest is compounded continuously.
  - How long will it take to reach a balance of \$4000 if interest is compounded continuously?

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